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The History of Maxwell's Equations

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James Clerk Maxwell was born in 1831 in Scotland to a family of Fellows of the Royal Society, an

elite organization of the top scientists of all disciplines in Great Britain (The Royal Society, 2011). Maxwell

began his academic career quite early. He presented his first paper "Oval Curves" to the Royal Society of

Edinburgh when he was fourteen (Forfar, 1995). Maxwell began his undergraduate studies at Edinburgh

University at age sixteen and entered graduate school at Cambridge University at age nineteen. After

graduation, he was a fellow and professor at a variety of colleges in the United Kingdom. Maxwell was

inducted as a Fellow of The Royal Society of Edinburgh when he was 25, and promoted to a Fellow of The

Royal Society at age 30. After a fruitful career, James Maxwell passed away at the age of 48 of stomach

cancer, which was oddly the same cause and timing of his mother's death when Maxwell was eight years

old (Forfar, 1995).

In 1819, twelve years before Maxwell was born, the field of electromagnetism was born. Hans

Christian Oersted of Denmark had suspected that electricity and magnetism were related for some time

and was finally able to create some evidence for it. In his experiment, Oersted held a wire above a

compass. When a current travelled through the wire, he noticed that the needle of the compass moved.

This launched an investigation of how electricity and magnetism were related. André Ampère was so

excited after learning of Oersted's discovery that he created a rough

mathematical theory that described it in one week in 1820, and

published his finished equation in 1821 (Peters, 2000). Ampère's

equation becomes the fourth of Maxwell's famous equations. A year

later, the first galvanometer was made, which was greatly beneficial   to

the measurement of electric current (The Encyclopedia Americana

Corporation, 1918). A galvanometer relies on the findings of Oersted's

experiment: when a current travels through a coil, a nearby

magnetized needle is deflected. As was found after Oersted, the needle
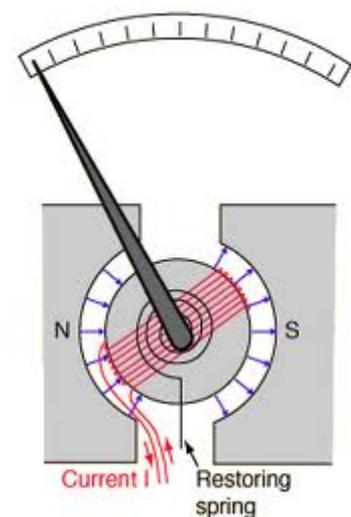
A galvanometer[1]

is deflected in proportion to the strength of the current (Florida State University, Los Alamos National

Laboratory, University of Florida, 2011).

The most productive scientist in electromagnetism before Maxwell was Michael Faraday, who was

actually a chemist. In the course of his research, which lasted from 1831 to 1855 (The Encyclopedia

Americana Corporation, 1918), he produced over 16,000 perfectly organized entries in his laboratory

notes (Peters, 2000). The results of his experiments formed the basis for Maxwell's electromagnetic

theory. Although Faraday had an amazing physical intuition that guided him through his experiments to

successfully create a nearly complete model of electromagnetic phenomena, since he was not at all a

mathematician, he was only able to describe this model in words and did not unify his results into a theory

(The Encyclopedia Americana Corporation, 1918). Since the physicists of the time were mathematically

trained, they hardly paid Faraday's results any attention. They saw the lack of mathematics as a lack of

sophistication (Hunt, 1983).

Although Faraday completed an amazing number of experiments, there are two series that are

most important to concretely understand his contributions to Maxwell's theory. The first set of

experiments allowed Faraday to discover the principle of electromagnetic induction. He wound two

separate wires around a wooden bobbin and insulated them from each other. To one he connected a

battery, and to the other he connected a galvanometer. He discovered that by breaking or reconnecting

the circuit connected to the battery, a current of the opposite direction was induced in the other wire.

This amazing discovery forms the basis of any device that uses electricity, including telephones, which

were invented later in the 19th century. Faraday also found that simply moving a magnet closer or farther

away from a closed circuit would induce a current (The Encyclopedia Americana Corporation, 1918). This

discovery allowed Faraday to design and build a generator as well as the first transformer and the first

electric motor. When asked by a politician what the use of this work was, Faraday replied "'at present, I

Iron filings[2]

don't know, but one day you will be able to tax them'" (Peters, 2000, p. 6). Electromagnetic induction is a truly incredible phenomenon that redefined how the sources of electricity were thought of. The second experiment is actually one that many elementary school children carry out in their classrooms. It involves spreading iron filings on a surface and simply placing a magnet on them. This causes the iron to be moved by the magnetic field in a pattern that loops from one end of the magnet to the other. Faraday saw this pattern and thought that these "lines of force," as he called them, must be the key to electromagnetism. When he induced an electric current in a wire with a magnet, he saw this line of force curving around the wire, as we now know is the case.

The first paper that James Maxwell published after earning his graduate degree was a 75 page analysis of Faraday's concept of "lines of force" published in 1855 and was suitably titled "On Faraday's Lines of Force" (Forfar, 1995). Maxwell translated Faraday's ideas into mathematics. Maxwell created vectors to describe the main players of electromagnetism: "**E**, the electric field intensity, **H**, the magnetic field intensity, **B**, the magnetic flux density, and **I** the electric current density. **E** and **H** are forces and **B** and **I** are fluxes (lines of force) produced by the forces" (Peters, 2000, p. 9). A way to picture flux is to imagine having a square loop of wire in a flowing river. The flux of the velocity of the water would be like considering how much water will flow through the loop. The flux of an electric field is proportional to the number of electric field lines that go through such a loop (Sciolla, 2004).

A square loop of wire in flowing water[3]
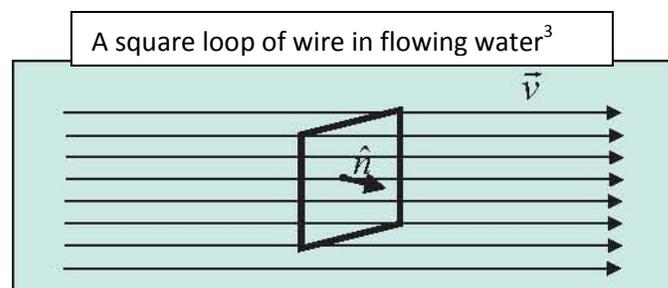
Notice that Maxwell had transformed Faraday's "lines of force" into mathematical concepts that we still use today. Fields are used in many disciplines across mathematics. The first mention of a field in Maxwell's A Treatise on Electricity and Magnetism was of an electric field, which he described as
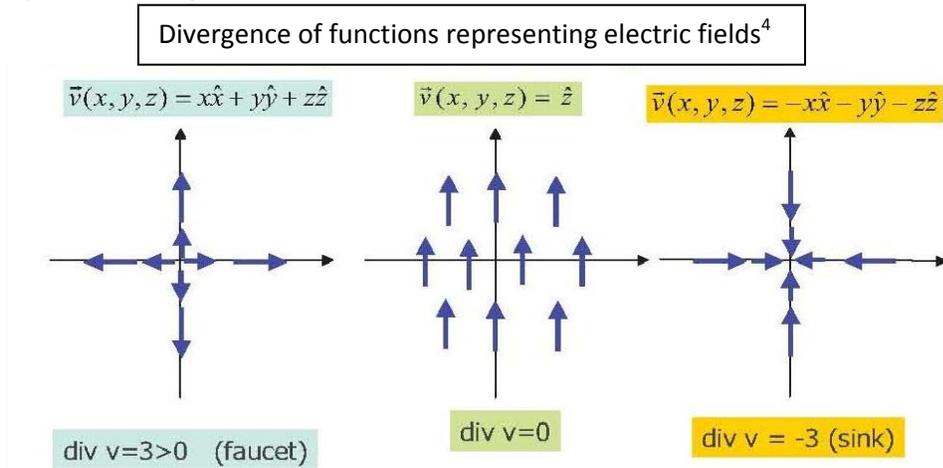
"the portion of space in the neighborhood of electrified bodies, considered with reference to

electric phenomena. It may be occupied by air or other bodies, or it may be a so-called vacuum,

from which we have withdrawn every substance which we can act upon with the means at our

disposal. If an electrified body be placed at any part of the electric field it will, in general, produce

a sensible disturbance in the electrification of the other bodies" (Maxwell, 1892, pp. 47-48).

With these vectors, Maxwell created countless equations to describe his electromagnetic theory (Maxwell,

1892). Despite the publishing of A Treatise on Electricity and Magnetism, which was a handbook starting

from square one for everything one could know about electromagnetism at Maxwell's time, not much

attention was paid to Maxwell and his revolutionary ideas. It was not until Heinrich Hertz found

experimental evidence for Maxwell's concept of a field in 1887 by discovering electromagnetic waves in

space did the scientific community begin to look at Maxwell's theory.

Before we explore the basic equations of Maxwell's theory of electromagnetism, we must learn

two operations. First recall that a derivative $\frac{df}{dx}$ of a function f is the proportion of the change in f(x) and

the change in x: $df = \frac{\partial f}{\partial x} dx$. The proportion $\frac{\partial f}{\partial x}$ is called a partial derivative because it is used when a

function is of more than one variable and is simply the derivative of that function with respect to x,

treating all other variables as constants. When a function is of more than one variable, such as f(x, y, z), we

have $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$. The del operator, ∇, is a pseudo vector of the partial derivatives of f:

$\nabla \equiv \frac{\partial}{\partial x} \vec{x} + \frac{\partial}{\partial y} \vec{y} + \frac{\partial}{\partial z} \vec{z}$ and is used in the two following operations (Sciolla, 2004).

The first operation is called divergence, and is denoted by $\nabla \cdot v \equiv \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$ where *v* is a

function of x, y, and z. This operation uses the dot product. Recall that the dot product of two vectors

$v = (u, w, r)$ and $q = (a, b, c)$ is $v \cdot q = ua + wb + rc$ and yields a scalar. The divergence of a function

represents how much a function "spreads around a point" (Sciolla, 2004). If $v(x, y, z)$ represents an
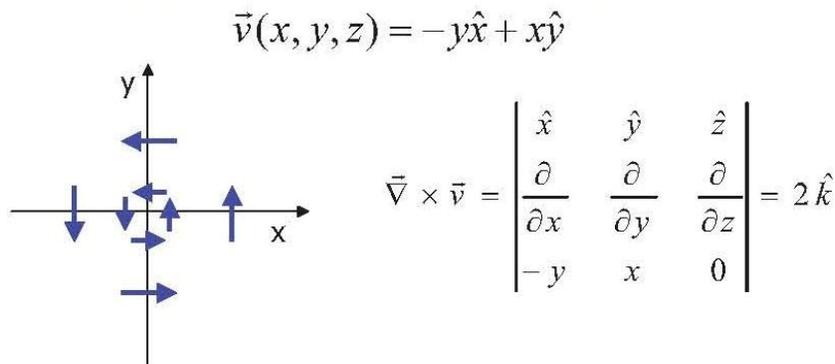
electric field, then the divergence of the first function below shows that any electrons in the field are

being repelled away from the origin (Sciolla, 2004).

Divergence of functions representing electric fields[4]

$$\vec{v}(x,y,z) = x\hat{x} + y\hat{y} + z\hat{z} \qquad \vec{v}(x,y,z) = \hat{z} \qquad \vec{v}(x,y,z) = -x\hat{x} - y\hat{y} - z\hat{z}$$

div v=3>0   (faucet)   div v=0   div v = -3 (sink)

The second operation is called curl, which describes how much a function "curls around a point,"

(Sciolla, 2004) and is denoted by $\nabla \times v$. An example of the calculation of the curl of a function

representing a magnetic field $v(x,y,z)$ is shown below. The determinant of the matrix below is a pseudo

cross product: $\nabla \times v = \hat{x}\left(\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}x\right) - \hat{y}\left(\frac{\partial}{\partial x}0 - \left(-\frac{\partial}{\partial z}y\right)\right) + \hat{z}\left(\frac{\partial}{\partial x}x - \left(-\frac{\partial}{\partial y}y\right)\right)$

$\nabla \times v = \hat{x}(0-0) - \hat{y}(0-(-0) + \hat{z}(1-(-1)) = 2\hat{z}$ which is sometimes written as $2\hat{k}$.

$$\vec{v}(x,y,z) = -y\hat{x} + x\hat{y}$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix} = 2\hat{k}$$

This is a vortex: non zero curl!

The curl of a function representing a magnetic field[5]

With these operations in mind, we are ready to look at the evolution of Maxwell's equations.

In 1861, Maxwell published the beginnings of his famous equations. It was while developing these equations that he discovered that light is similar to electricity and magnetism. Since these equations were of a different form than we see them today, to better appreciate Maxwell's distillation of Faraday's theory, we will look at the form in which mathematician and physicist Oliver Heaviside wrote them a couple of decades after the publication of Maxwell's treatise. Two are most notable (Darrigol, 2005):

$$\nabla \times \boldsymbol{E} = \boldsymbol{M} - D\boldsymbol{B}/Dt$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + D\boldsymbol{D}/Dt$$

where **M** is the magnetic current, which describes how a magnetic field seems to flow from one pole to the other, **j** is the electrical current, and $D\boldsymbol{D}/Dt$ is the displacement current, which is basically a potential for electric current to flow. The remaining variables are described above.

Electric potential was a new concept that grew to change electromagnetic theory completely. It was Faraday that had first thought of this paradigm but could not express it mathematically (Peters, 2000). Maxwell actually created a new form of derivative in order to achieve this expression: D**Y**/Dt is the convective derivative defined by D**Y**/Dt = ∂**Y**/∂t −∇×(**v**×**Y**) + **v**(∇·**Y**), where **v** is the velocity of "the states" (Darrigol, 2005, p. 2) which are **E**, **H**, **D**, and **B**. Since **D** is electric flux, D**D**/Dt represents varying electric current potential. This and D**B**/Dt were added by Maxwell to the original equations written by Heinrich Lenz (the first) and André Ampère (the second) (Darrigol, 2005, p. 2).

A Treatise on Electricity and Magnetism, published in 1873, records the transformation of Maxwell's complete theory of electromagnetism into twelve equations (Maxwell, 1892). It is an amazing feat for one scientist to write the equations that summarized an entire field of research. The self-taught mathematician and physicist Oliver Heaviside, also from Great Britain, further condensed Maxwell's theory into the four equations that are most known today: (Heaviside, 1893).

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{D\boldsymbol{B}}{Dt}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + D\boldsymbol{D}/Dt$$

The first equation relates electric flux to the volumetric charge density, ρ. The second equation says that a magnetic field does not diverge because there are no monopoles (Sciolla, 2004). The third equation summarizes the result from Faraday's electromagnetic induction: "the voltage induced in a circuit was proportional to the time rate of change of magnetic flux through the circuit" (Peters, 2000, p. 9). The fourth is from Ampère: "the magnetic field in a closed loop around a current was, in fact, equal to the current through the loop" (Peters, 2000, p. 9).

The modern partial derivative version of these equations is very similar, but constants have been added to some terms in order to conform to standard systems of unit. Here we will look at the version in cgs (centimeters, grams, seconds- cgs was actually first introduced a year after the publication of Maxwell's treatise (Wolfram Research Products, 2007)):

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}$$

where c is the speed of light (Sciolla, 2004).

Today, we also sometimes find it convenient to use Maxwell's equations in their integral form. The following are in SI units (Fowler, 2009):

$$\int E \cdot dA = q/\varepsilon_0$$

$$\int B \cdot dA = 0$$

$$\oint E \cdot dl = -\frac{d}{dt}(\int B \cdot dA)$$

$$\oint B \cdot dl = \mu_0(I + \frac{d}{dt}(\varepsilon_0 \cdot \int E \cdot dA))$$

where q is the electric charge, ε_0 is the electric constant, and μ_0 is the magnetic constant. The first equation says that "the integral of the outgoing electric field over an area enclosing a volume equals the total charge inside" (Fowler, 2009). The second is still another way of saying that there are no magnetic monopoles. The third and fourth equations have a path integral that simply denotes integration around a closed path. In this case, that path is usually a wire. The left side of the equation "gives the total voltage change around the circuit, which is generated by a varying magnetic field threading through the circuit" (Fowler, 2009). The fourth equation "gives the total magnetic force around a circuit in terms of the current through the circuit, plus any varying electric field through the circuit (that's the displacement current)" (Fowler, 2009).

Maxwell's equations are amazing because they summarize the fundamental phenomena related to electromagnetism in four concise equations. However, his equations had more of an effect on electromagnetism than additional convenience. Maxwell extended, connected and translated Faraday's concepts of electromagnetism into a full-fledged mathematical theory, which put it on the table for comparison to the leading electromagnetic theory at the time. We will compare these theories using the

Leyden Jar, a device still used in Maxwell's time, as pictured. The

Leyden Jar has two coatings of a conductive metal that do not

touch each other, one on the inside of an insulating glass, and the

other on the outside. The inner coating is charged with a metal rod

and then the metal rod is removed. When a scientist touches the

outside of the jar, the scientist receives an electric shock (Wolfram

Research Products, 2007). This surprised scientists because they

The Leyden Jar[6]

thought only the inner coating had been charged. The differing explanations of this phenomenon

encompass the heart of the difference between Maxwell's theory and the leading electromagnetic theory

(The Encyclopedia Americana Corporation, 1918).
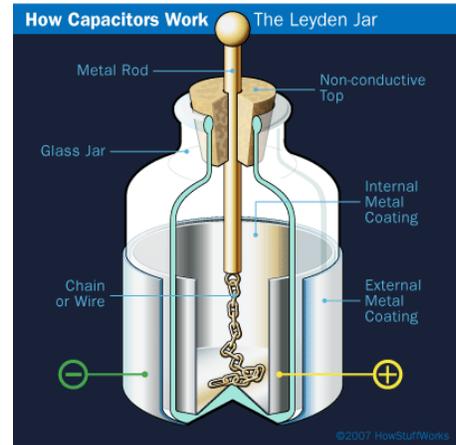
This prevalent theory was called "action at a distance." This theory assumes that insulators

completely block electric current. Therefore, the reason that a charge could be received from the outer

coating is that electricity works at a distance, as gravity does. While electricity was building up on the

inner coating, an electricity of an opposite kind was attracted to the outer coating. Therefore, when

touched, the outer coating could deliver a shock (The Encyclopedia Americana Corporation, 1918).

Maxwell saw the phenomenon of the Leyden Jar and was inspired by Faraday's "lines of force."

Maxwell expanded upon Faraday's conception of electricity to assert that not only does electricity travel

along lines of force, but also requires a medium. He saw insulators such as glass as being resistive to

electric current, not immune to it. He saw the process of charging the inner coating to be like displacing

electricity in the way that you would displace a spring by stretching it away from its equilibrium. The

resistive force of the glass blocks the electricity from returning to its equilibrium until the charging force

overcomes it. In the spring example, the charging force would be the force of the spring and the resistive

force would be the force of the hands that are pulling the spring apart. When the force of the spring

overcomes the force exerted by the hands, the spring snaps back into equilibrium. This is how Maxwell

saw the phenomenon of the Leyden jar: when the charging force overcomes the resistive force, the

electricity snaps back into equilibrium through an electric current (The Encyclopedia Americana

Corporation, 1918). This change in conception would greatly simplify the mathematics involved in the

study of electromagnetism as well as more accurately explain electromagnetic phenomena (Darrigol,

2005).

It is amazing how quickly electromagnetism developed in such a short time from its beginnings

with Oersted's experiment in 1819. However, this quick pace was not just coincidence, but was related to

the cultural changes that Great Britain was going through during the 19th century. During the mid 19th

century, the idea that the principle of free trade would bring peace and prosperity began to percolate

through European society. For some time, the dominating economic philosophy had been mercantilism,

which was a doctrine of strict protection of a state's resources, high tariffs on trade, and colonial

domination. These ideas of the benefits of free trade in the intellectual culture had a place in the scientific

culture as well. Rather than working independently on one's reputation and theory, scientists began to see

the benefits of developing upon their peers' ideas and discussing the relative merit of scientific theories as

a community (Turner, 1980). Maxwell spent the majority of his work in electromagnetism simply

converting Faraday's theory into mathematics, but the combination of their respective talents created an

incredible contribution to science. It was this idea of free trade that made Maxwell want to share his

theory with Europe, not just to dominate the field, but to peacefully offer a comparison to the leading

theory (Forfar, 1995). During the mid 19th century, the British scientific culture also fully equated society's

progress with scientific progress. They had seen science create the telegraph, seen the telegraph

revolutionize communication and European society, and seen the scientific advancement of the telegraph

continue to revolutionize society. They saw the power of science to change the world (Turner, 1980). For

physicists specifically, it was the power of the telegraph that was a major motivation to understand

electromagnetism completely to advance communication (Maxwell, 1892).

It was because of this belief that scientific advancement was important for society's progress that

scientists began wanting to influence public decisions and have public funding. However, the people who

were in the public forefront at the time, politicians and businessmen, dismissed the idea that scientific

work had anything to do with their work. In an attempt to win society over, many scientists began to

emphasize how important science was to the security, economic competency, and stability of Great

Britain. This strategy was drawing on the fact that British science and British politics and business had a

common enemy: Germany. By the late 19th century, Germany became an industrial threat to Great

Britain's economic dominance, and was slowly becoming a military threat as well. Germany had also been

a thorn in the sides of British scientists for years. Germany's public funding and public recognition of their

scientists' research had made German science flourish in the 19th century, and was much more influential

and prosperous than British science (Turner, 1980). In fact, the leading theory of electromagnetism,

"action at a distance," was developed in Germany and it was these German scientists who looked down

upon and refused to acknowledge Maxwell's theory. Frustrated with the lack of response of the scientific

community to his paper "On Faraday's Lines of Force," Maxwell transformed his comprehensive theory

into twelve equations, a great feat even for today's pace of scientific progress (Forfar, 1995). However, the

German scientists still refused to relinquish their monopoly on electromagnetism, and simply dismissed

Maxwell's theory as something that just works out on paper (Turner, 1980). Oliver Heaviside's editing of

Maxwell's equations began to turn the tide, but it was not until Heinrich Hertz, a German physicist,

demonstrated in 1887 the existence of a field of electromagnetism by finding electromagnetic waves in

space that German scientists began to accept Maxwell's theory (Hunt, 1983).

Maxwell's theory of electromagnetism was revolutionary. It completely changed the study of electromagnetism as well as European society. After Hertz discovered electromagnetic waves in space, the wireless telegraph was invented, which made fast communication possible across oceans for the first time. Maxwell's theory also made long-distance electrical wires effective, something the "action at a distance" theory was not able to do (Hunt, 1983). Still today, we owe an enormous amount of technology to Maxwell's theory of electromagnetism and his perseverance through cultural obstacles to advocate his talent of mathematical interpretation.

References

Darrigol, O. (2005). *The Genisis of the Theory of Relativity.* Retrieved October 12, 2011, from Séminaire Poincaré: http://www.bourbaphy.fr/darrigol2.pdf

Florida State University, Los Alamos National Laboratory, University of Florida. (2011). *Galvanometer*. Retrieved December 6, 2011, from Magnet Lab: National High Magnetic Field Laboratory: http://www.magnet.fsu.edu/education/tutorials/java/galvanometer/index.html

Forfar, D. O. (1995, July). *James Clerk Maxwell: Maker of Waves.* Retrieved November 30, 2011, from James Clerk Maxwell Foundation: http://www.clerkmaxwellfoundation.org/Maker_of_Waves.pdf

Fowler, M. (2009, May). *Maxwell's Equations and Electromagnetic Waves*. Retrieved November 30, 2011, from Galileo and Einstein: http://galileo.phys.virginia.edu/classes/109N/more_stuff/Maxwell_Eq.html

Heaviside, O. (1893). *Electromagnetic Theory.* Boston: "The Electrician" printing and publishing company, limited.

Hunt, B. J. (1983, September). "Practice vs. Theory": The British Electrical Debate, 1888-1891. *Isis, 74*(3), 341-355.

Maxwell, J. C. (1892). *A Treatise on Electricity and Magnetism.* Oxford: Clarendon Press.

Peters, R. A. (2000, April 5). *A Brief Outline of the History of Electromagnetism.* Retrieved October 12, 2011, from Richard Alan Peters II, Ph.D.: http://www.vuse.vanderbilt.edu/~rap2/papers/em_history.pdf

Sciolla, G. (2004). *Physics II: Electricity and Magnetism.* Retrieved October 7, 2011, from MIT OpenCourseWare: http://ocw.mit.edu/courses/physics/8-022-physics-ii-electricity-and-magnetism-fall-2004/

The Encyclopedia Americana Corporation. (1918). Electricity, Its History and Progress. In *The Encyclopedia Americana* (Vol. 10, pp. 175-180). New York: J. B. Lyon Company. Retrieved November 2011, from http://books.google.com/books?id=62UMAAAAYAAJ

The Royal Society. (2011). *Fellowship*. Retrieved December 2, 2011, from The Royal Society: Excellence in Science: http://royalsociety.org/about-us/fellowship/

Turner, F. M. (1980, December). Public Science in Britain, 1880-1919. *Isis, 71*(4), 589-608.

Walter Lewin, J. B. (2010). *Physics II: Electricity and Magnetism.* Retrieved October 7, 2011, from MIT OpenCourseWare: http://ocw.mit.edu/courses/physics/8-02sc-physics-ii-electricity-and-magnetism-fall-2010/

Wolfram Research Products. (2007). *cgs*. Retrieved December 7, 2011, from Eric Weisstein's World of
        Physics: http://scienceworld.wolfram.com/physics/cgs.html

Wolfram Research Products. (2007). *Leyden Jar*. Retrieved December 7, 2011, from Eric Weisstein's World
        of Physics: http://scienceworld.wolfram.com/physics/LeydenJar.html

Endnotes

1. http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/galvan.html

2. http://www.cyberphysics.co.uk/My_Notes/Magnets1.htm

3. http://ocw.mit.edu/courses/physics/8-022-physics-ii-electricity-and-magnetism-fall-2004/lecture-notes/lecture2.pdf

4. http://ocw.mit.edu/courses/physics/8-022-physics-ii-electricity-and-magnetism-fall-2004/lecture-notes/lecture1.pdf

5. http://ocw.mit.edu/courses/physics/8-022-physics-ii-electricity-and-magnetism-fall-2004/lecture-notes/lecture1.pdf

6. http://electronics.howstuffworks.com/capacitor.htm/printable