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John Allen Paulos, Innumeracy: Mathematical Illiteracy and its Consequences

Louis D. Grey

BOOK REVIEWS

John Allen Paulos, *Innumeracy: Mathematical Illiteracy and its Consequences*. New York: Hill & Wang, 1988. 135 pp. $16.95.

by Louis D. Grey

Most of us are aware of the literacy problem which exists in this country and throughout the world. According to the U.S. Department of Education, 1 in 5 American adults is functionally illiterate — 20% of the total adult population in this country. In addition, another 34% of American adults are only marginally literate. And what is perhaps a more alarming statistic is that 2.2 million people are added to the adult illiterate population each year. Illiteracy is on the increase in the United states.

Being functionally illiterate is an enormous handicap which is not easily hidden, especially in an increasingly technological society. Individuals afflicted with this malaise are usually confined to the lower rungs of the economic and social ladder. They are culturally deprived and the things which most of us enjoy and take for granted are beyond their capabilities. They certainly have little hope of participating in any decision-making process or contributing to alleviating the social problems of the day. There are some notable exceptions, however, people who have managed to "fake" it for years, until some traumatic event exposes the problem and forces them to seek help. In a University community dedicated to scholarship and learning, whose very existence is predicated on these basic skills, illiteracy is a phenomena that is often difficult to comprehend. Fortunately we recognize that such a problem exists, that it is serious and must be corrected. Support groups have been organized and organizations such as literacy volunteers have begun to make progress in alleviating this problem.

But as important as it is, illiteracy is not our subject here. John Allen Paulos, Professor of Mathematics at Temple University, has written a book called *Innumeracy*. Few mathematics books make the best-seller lists, but this one has received a good deal of media attention because it deals with a problem which, like illiteracy, afflicts millions of our citizens. The purpose of Professor Paulos' book is to discuss this problem and point out the consequences not only to the individuals but to society at large.

By "innumeracy," Paulos means "an inability to deal comfortably with the fundamental notions of number and chance." In short, Paulos is talking about the problem of mathematical illiteracy, a problem which is not confined to uneducated people, but one which afflicts people who are otherwise very knowledgeable. Mathematics teachers at any college or university are exposed to this problem on an almost daily basis. Our students cannot deal with fractions, percentages, or simple algebraic concepts and some of them have been known to become physically ill if asked to solve a "number problem."

As a result of this, we devote a good deal of our resources to placement testing and remedial work and our students limp along when they should be running. But the problem is not confined to mathematics students. It is far more extensive. A distinguishing characteristic of this problem is that people are often proud to acknowledge it. To say "I hated math," "I can't even balance my checkbook" or "I'm a people person, not a numbers person" is for some people a badge worn with pride, almost flaunted whenever the occasion arises. One reason that it is flaunted, as Paulos points out, is because the consequences are not usually as obvious as those of other weaknesses, and this in essence is what the book is all about.

The book looks at real-world examples of innumeracy, involving a wide range of topics such as stock scams, choice of a spouse, newspaper psychics, diet and medical claims, the risk of terrorism, astrology, sports records, elections, sex discrimination, UFOs, insurance and law, psychoanalysis, para-psychology, lotteries, and drug testing. By applying elementary ideas from probability and statistics which depend on nothing more than common sense and arithmetic, Paulos succeeds in debunking a number of common misconceptions without adopting the scolding tone associated with enterprises of this type. Paulos is concerned but not angry and this is reflected in his writing. He is also not devoid of humor, as will be seen from many of the examples.

A glance through the book will convince anyone that there is very little here in the way of formal mathematics to frighten the reader. There are practically no equations, no strange looking symbols, no mathematical jargon or anything else to set the book apart from what one usually expects a book to look like. The book is relatively short and can easily be read in a few hours. It would probably be best, however, to ponder over the examples in a leisurely fashion and take whatever time is necessary to understand the author's point. No one need worry about an examination.

The strength of the book, as I have already said, lies in the examples. They are carefully thought out and not contrived merely so that the author can make a point. Practically everyone will find at least a few examples which relate to situations that they will have en-countered themselves. The style is leisurely and gentle and the reader will not feel that the author's conclusions have been forced upon him or her.

Mathematical arguments are best illustrated by examples. I have selected three which are more or less typical of what can be found in the book. The first example deals with the phenomenon of coincidence. Practically everyone has had an experience where events have occurred that seemed so unlikely that we could not attribute them to chance or coincidence. We simply have an intuitive feeling that some other force is at work and that the probability that the event is a chance event is infinitesimally small. Suppose you were introduced to a total stranger and as a result of striking up a conversation with him, you discovered that his sister was a classmate of your spouse. It turns out this sort of coincidence is really surprisingly common. Most of us would feel that it runs totally counter to our intuition. We are inclined to think that a relatively rare event has occurred.

Let us look at what has taken place. In this case there are two intermediaries linking you and the stranger. You know your spouse who knows his sister who knows him. Suppose we now pose the following problem: How many intermediaries on average link any two people? Making a few reasonable assumptions, it can be shown that the number of intermediaries is relatively small. In other words, it is likely that only a few intermediaries link you with almost anyone you care to name. I will not prove this here, but suffice it to say that the proof does not involve any terribly complicated mathematics and even a high school student should be able to understand it without any difficulty.

The second example I want to discuss deals with the subject of medical tests. While I use cancer as an illustration, the argument is general and could apply equally well, say, to AIDS or any of the other well-known diseases. For argument's sake, suppose a test for cancer is such that 98% of the time when the test is positive, the person being tested does have cancer, and 98% of the time a person being tested who does not have cancer will test negative. Assume further that on average, one person in two hundred or .5% of the population has cancer. Put yourself in the position of a person who has been tested and told that the test was positive. Should you now be very depressed? Should you start winding down your life? Most of us would be inclined to say yes, assuming the information just given is true.

But let's take a closer look at the problem. Suppose that 10,000 tests for cancer have been administered. We expect on average 50 of these 10,000 people (.5% of 10,000) to have cancer, and since 98% of these will test positive, we would expect 49 positive tests. Of the remaining 9,950 people who do not have cancer, 2% of them will still test positive due to the inaccuracy of the test, and this will yield 199 positive tests (.02 times 9,950). Thus we have a total of 248 positive tests (49 + 199). Now most of these positive tests are clearly false. Hence the conditional probability of having cancer given that you have tested positive is only 49/248 or about 20%. The answer to the questions posed before is that you should be cautiously optimistic since on the basis of the test alone, the odds are about 4 to 1 in your favor.

As Paulos points out, this unexpected result with a test that is 98% accurate should give legislators pause when they contemplate mandatory tasting for AIDS or whatever. Consider also that many tests are far less accurate. The PAP test for cervical cancer which may only be 75% accurate would lead to an even more optimistic result. So the con-clusion to be drawn is that any test which produces a large number of false positives may lead to inferences that are totality unwarranted. Thus, with an inaccurate test such as a lie detector test, truthful people who fail the test will far outnumber the liars.

My third and final example deals with the subject of predictive dreams. If you dreamt that you were going to win the lottery and you actually won, you would most likely believe that some form of precognition had occurred rather than a chance event. Now suppose we ask what the probability is of having a predictive dream, e.g., one which either comes true or is at least matched in some dramatic detail. Suppose for argument's sake that the probability of such a dream is 1/10,000. Another way of looking at this is to say that the probability of a non-predictive dream is 9,999/10,000. If we assume that whether a dream matches experience or not is independent from day to day, then the probability of two successive non-matching dreams is given by (9,999 x 9,999)/(10,000 x 10,000). For a years worth of non-predictive dreams the probability is about .964. But this means that among those who dream every night, about 1 minus .964 or .036 (that is, 3.6%) will have a predictive dream. Given that there are large numbers of people who dream every night, this translates into large numbers of people who will have predictive dreams. Even if the probability of a predictive dream is assumed to be much lower (e.g., one in a million), this will still translate into large numbers of people who have predictive dreams by chance alone.

Some readers may still feel uncomfortable with the arguments that are advanced by Professor Paulos since the conclusion again is so counter-intuitive. But what he is saying is that in these instances, our intuition is an unreliable guide. And even if one feels uncomfortable, we cannot accuse Paulos of indulging in any fancy mathematics since he uses only the most elementary notions of arithmetic and probability theory.

There are many more such examples in the book, some serious and some rather amusing. But they all share one common element. If we are uncomfortable with the reasoning, if we cannot understand it, much less do it for ourselves, then we show ourselves to be victims of "innumeracy," and as with all victims there is a price to be paid. Furthermore, when the number of victims is large, as it very well is in this case, then there may also be a huge debt which society is obligated to pay. Professor Paulos does not tell us how to correct the problem, and provides little in the way of sage advice on this subject. But after all, that was not the purpose of the book. What you will find here is a charming collection of well-thought out examples depicting situations in which "innumeracy" arises and pointing out its consequences. And despite the underlying seriousness of the topic, it is fun to read.