Hedging with Volatility

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Abstract

A risk-averse investor with a long equity position is presumably interested in identifying a hedging strategy that protects the value of that investment. The common approach encompasses using either financial derivatives or holding assets (such as gold or Swiss francs) as portfolio hedges as they show negative correlation with equities. This paper proposes using volatility indexes as portfolio hedges instead; it shows that a volatility-based dynamic hedging strategy is the most effective at protecting the value of an equity investment.

Keywords: Minimum Variance Portfolio; Portfolio Rebalancing; Sharpe Ratio

1. Introduction

The objective of this paper is to develop a hedging strategy for an equity investment.

I consider two equity investments: one in the S&P 500 index and another one in the DAX 30 index. Different assets are considered as potential equity hedges and their hedging properties are examined by performing both in-sample and out-of-sample tests.

I start by showing that the correlation between the returns of the equity investment and the returns of the volatility assets is significantly negative. Next a linear regression model is used to verify the significance of the historical correlation results - results show that the volatility assets (VIX and VDAX) are strongest hedges for an equity investment.

Since the regression results highlight the superior hedging properties of volatility assets, I then test the performance of a portfolio which includes them as equity hedges. The in-sample portfolio optimization results are consistent for both investments and show that the portfolios which include the VIX and the VDAX have a better performance than all the other portfolios. The out-of-sample portfolio optimization results confirm that portfolios that include volatility assets perform better than all other portfolios because adding those assets significantly reduces risk.

Hedging has the potential of providing an offset against losses due to market turbulence. Therefore, developing a hedging strategy for an equity investment is relevant because as risk-averse investors enter into long equity positions they are keen to identify hedging strategies to protect the value of their investments.

A common strategy is to hedge by using derivative contracts. Another strategy is to hold assets which returns show historical negative correlation against the returns of an equity investment. Assets such as government bonds, certain commodities such as gold and certain currencies such as Swiss Francs have that correlation and are frequently used as portfolio hedges. A different strategy is to hold volatility assets since their returns are also negatively correlated with the equity returns. On my paper I investigate the hedging strength and effectiveness of all these assets.

I show that investors should consider a volatility-based dynamic hedging strategy because this strategy outperforms a strategy that relies on non-volatility assets as equity hedges. Furthermore I show that this strategy does not require frequent rebalancing because the portfolio optimization results indicate that the portfolios weights do not change significantly over time. In addition, I also show that the strategy maintains its effectiveness even under market stress.

My contribution to the body of work on risk management is two-folded: I develop a multiregional volatility-based dynamic hedging strategy based on the expected returns and variance/covariance matrix; and I assess the hedging strategy effectiveness under normal and turbulent markets.

The paper is organized as follows: section 2, reviews the relevant literature regarding both portfolio hedging and portfolio optimization; section 3, presents the data and methodology used in the paper;

section 4, presents the results for all the in sample and out of sample statistical tests; and section 5, presents the conclusion. All statistical results all presented on the Appendix.

2. Literature Review

Commodities are one of the assets used as equity hedges. The attractiveness of commodities to investors is that it allows them to hold a physical commodity that has inherent value, which holds its value when equities decline and that can be bought and sold around the world – gold has these characteristics and that is why it is attractive to investors. On a study of U.S., U.K. and German stock, bond and gold returns, Baur, and Lucey (2010) found that gold is a hedge against stocks on average and a safe-haven in extreme stock market conditions. Nevertheless, they also provide evidence that gold's safe-haven property is short-lived i.e., investors that hold gold more than 15 trading days after an extreme negative shock lose money with their gold investment - this suggests that investors buy gold on days of extreme negative returns and sell it when market participants regain confidence and volatility is lower. Hillier, Draper and Faff (2006) show that gold, platinum and silver have low correlations with stock index returns indicating potential diversification benefits within broad investment portfolios. Furthermore, their paper also shows that the three precious metals can serve as hedging instruments especially during periods of abnormal stock market volatility. Jaffe (1989) showed that adding gold to a diversified portfolio increases the portfolio average return while reducing its standard deviation. On a study about the benefits of adding precious metals to a U.S. equity portfolio, Conover et al. (2009) show that adding the commodities to the equity portfolio improves portfolio performance and also provides protection against inflation.

The Swiss franc is also used as a hedge for an equity portfolio. Campbell et al. (2010) show that over a 30 years period (1975-2005) the U.S. dollar, the euro and the Swiss franc moved against world equity markets and can be therefore very attractive hedges for global equity investors. On a more recent paper, Lee (2017) identifies the Swiss franc as one of six potential safe-haven currencies (the others being the Japanese yen, British pound, euro, Canadian dollar and Norwegian krone). He shows that the Swiss franc (and the Japanese yen) is negatively related to risky assets and that the negative relation is stronger in times of crisis than in times of growth – which in turn qualifies the currency as a strong safe-haven.

Recently, volatility indexes such as the CBOE VIX have been proposed as hedging instruments. Hood, and Malik (2013) evaluate the role of both metals (gold included) and volatility (in the form of VIX) as portfolio hedges. They find that gold serves as a hedge and a weak safe haven for an equity investment in the US stock market. But more importantly they show that 1) VIX performs better than gold as portfolio hedge i.e., VIX is a stronger hedge and stronger safe haven and 2) in periods of extremely low or high volatility, VIX maintains the negative correlation with the US stock market while that negative correlation ceases to exist with gold. Although the negative correlation between volatility indexes and equity returns is well understood and documented, not much research has been done in studying its portfolio hedging effectiveness.

Although conventional hedging strategies are effective at hedging equity portfolios, it is not clear whether they optimize the portfolio risk/return for investors. I propose a strategy that identifies volatility assets as hedges for an equity portfolio and examine the effectiveness of a dynamic hedging strategy. With a dynamic hedging strategy, the investor hedges the equity investment by 1) forming a portfolio consisting of positions in the equity asset and in a risky asset and 2) b changing the portfolio weights periodically given the dynamic nature of the strategy. In this strategy it becomes critical to estimate how much it will cost the investor to rebalance the portfolio and to factor that cost in the assessment of the hedging strategy overall effectiveness. Rather than trying to model the cost of rebalancing an equity portfolio and coming up with an arbitrary generated number, I focus instead in examining how the portfolio weights change over time. Material changes in the portfolio weights, imply frequent rebalancing; minimal or no changes in the portfolio weights, imply no need for rebalancing.

I regress the returns of the equity investment against the returns of the assets in order to evaluate the significance of the historical correlation results. In other words, I'm interested in assessing the strength of each asset as an equity hedge. A negative regression coefficient indicates that the asset is a strong hedge while a positive or neutral regression coefficient indicates that the asset is a weak hedge. The regression model I use is based on the work of Baur and McDermott (2010).

3. Data and Methodology

I consider two equity investments: a U.S. investment and a European investment. The investments are in equity indexes: the S&P 500 and the DAX 30. The data sample (daily observations) for the US investment ranges from January 1, 1990 to December 31, 2016 while the data sample for the European investment ranges from January 1, 1999 to December 31, 2016[1]. I consider the following assets as equity hedges: CBOE VIX, VDAX-New, CHF/USD exchange rate, CHF/EUR exchange rate, West Texas Intermediate, Brent-Europe, Copper, Aluminum, Platinum and Gold. These assets represent three different categories: volatility assets, currency assets and commodity assets. The use of options or other derivatives to hedge an equity investment is out of scope as I solely rely on the returns of the assets. All definitions, specific data sources and asset categories are summarized on Table 1 and 2 in the Appendix.

The descriptive statistics for all assets are summarized on Table 3. For the US investment, the historical annual average return is 8.71% and the annual standard deviation is 17.84% for the sample period. This compares against an annual average return of 13.75% and an annual standard deviation of 34.63% for all other assets combined. As for the European investment, the historical annual average return is 7.60% and the annual standard deviation is 24.40% for the sample period. This compares against an historical annual average return of 13.91% and an annual standard

[1] Since both my data samples are based on large time-series, I performed the Durbin-Watson test and verified that the residuals from the linear regressions are not auto-correlated.

deviation of 32.74% for all other assets. In conclusion, both investments have a similar risk/return profile.

While an investor can get a basic idea about the assets historical risk and return, from a hedging perspective, the investor is really interested in evaluating the hedging properties of those assets. It is then critical to investigate how the returns of the equity investment correlate with the returns of the different assets. This is an important hedging property because it provides the protection an investor needs for when the returns on the investment decrease. Essentially, a negative correlation indicates that returns of both variables move in opposite direction while a positive correlation indicates the opposite. Table 4 presents the correlation results for both investments: the returns on the S&P 500 and the returns on the DAX 30 are negatively correlated with the returns on the VIX and the returns on the VDAX. Furthermore, these two assets also show the highest negative correlation results with the respective equity investments.

Now that the investor knows which assets show the highest correlation with the equity investments, I then evaluate the strength of my assets as equity hedges. For that purpose, I model the relationship between the assets returns and the equity investment returns by using multiple linear regression. This allows me to calculate the regression coefficients and determine how significant is the correlation between the equity investment returns and the returns of the different assets. I use the ordinary least squares (OLS) regression method for estimating the regression coefficients. The econometric model is the following,

$$\mathrm{r}_{\mathrm{asset,i}} = \alpha + \beta_n \mathrm{r}_{\mathrm{investment,n,i}} + \varepsilon_{\mathrm{t}}$$

If the predicted regression coefficient is zero or positive, then the asset is a weak hedge and it will not protect the value of the equity investment; if the regression coefficient is negative, then the asset is a strong hedge and it will protect the value of the equity investment.

Hedge assessment results are summarized on Table 5. Regarding the US investment, the regression coefficients are negative and statistically significant for VIX and Swiss francs. This indicates that when the returns on the equity investment decrease, the returns of the VIX and Swiss francs increase therefore providing the desired protection for the investor. VIX’s regression coefficient is greater (-4.0924) than the one for Swiss francs (-0.0478) which indicates that VIX is a stronger hedge than Swiss francs. The regression coefficient results for all other assets are positive making them weak hedges. When it comes to the European investment, the regression coefficients are negative and statistically significant for VDAX, Swiss francs and gold. This indicates that when the returns on the equity investment decrease, the returns of these three assets increase therefore providing the desired protection for the investor. VDAX’s regression coefficient is greater (-2.4860) than the regression coefficient of Swiss francs (-0.0482) and gold (-0.0182) indicating that the volatility index is a stronger hedge than the other two assets. The regression coefficient results for all the other assets are either positive or not statistically significant making them, similar to the US investment, weak hedges. In conclusion, results are consistent across the two investments and clearly show that both VIX and VDAX are strong hedges for an equity investment and are the strongest among all other assets.

I then perform in-sample portfolio optimization tests. I start by calculating the excess returns of each asset. I use excess returns for practical reasons as it simplifies the portfolio optimization procedures. I create the optimal portfolios utilizing Markowitz portfolio optimization model. I set the portfolio optimization process to minimize the portfolio variance for a specific level of expected return. The optimal portfolio is the one that maximizes the portfolio Sharpe ratio. I calculate the optimal weights of each portfolio.

Let r_t denote the vector of the risky asset daily returns realized at time $t,$ and let $\mu_t = E_t[r_{t+1}]$ and $\Sigma_t = E_t[(r_{t+1} - \mu_t)(r_{t+1} - \mu_t)']$ denote the conditional expectation vector and covariance matrix r_{t+1}. Let r_t^f denote the return on the riskless asset (known in advance). The portfolio that achieves the target expected return μ_p with smallest standard deviation is found by solving,

$$\min w_t' \Sigma_t w_t \qquad \text{s.t.} \qquad w_t'\mu_t + (1 - w_t'1)r_t^f = \mu_p \tag{1}$$

The solution is well-known; the weights on the risky assets are,

$$w_t = \frac{(\mu_p - r_t^f)\Sigma_t^{-1}(\mu_t - r_t^f)}{(\mu_t - r_t^f)'\Sigma_t^{-1}(\mu_t - r_t^f)} \tag{2}$$

and the weight on the riskless asset is 1 - $w_t'1$.

The optimal weights depend on the conditional expectations μ_t and Σ_t. The expected return μ_t is notoriously difficult to estimate and therefore I use the simple historical average return as my estimate. Similarly, I use the simple yet consistent historical estimator of Σ_t. The use of these simple estimates should provide a conservative assessment of the performance of the optimal portfolio. If I was to use more sophisticated estimates, then presumably the performance would be even better.

In order to evaluate the hedging strategy effectiveness under turbulent market conditions, I regress the investment returns against the returns of each asset for the 2.5% worst quantile. In addition, I also calculate new optimal portfolios for the worst quantile and show their performance.

In the next section I use an out-of-sample methodology and perform portfolio optimization tests. I calculate the next day return of each portfolio on a sequential basis. I start by defining the in-sample data range as the basis for the out-of-sample tests. Rather than select a specific number of years, I use 33% of the total observations as in-sample data. The data sample for the US investment ranges from January 3rd 1990 to November 4th 1998 (2250 daily observations) and for the European investment from January 5th 1999 to December 13th 2004 (1494 daily observations).

I calculate the minimum variance portfolios. Then I add the next day observation to this data sample – by using an expanding window approach – and calculate the next day minimum variance portfolio and corresponding statistics (return, risk and Sharpe ratio). I define the annual target excess return as 5.7% as this is the excess return of each investment. As new observations are added to the original sample, the variance/covariance matrix is recalculated. I repeat the same procedure 4568 times and 3034 times for the US and European investments respectively. Essentially, with this procedure I re-examine the performance of each portfolio (i.e., calculate a new Sharpe ratio every day) by sequentially expanding the data sample. Ultimately, the objective is to determine which portfolio has the highest Sharpe ratio and whether this result is better than the Sharpe ratio of the unhedged equity investment.

After identifying which portfolio has the highest Sharpe ratio I investigate whether this hedging strategy requires rebalancing i.e., how often must the investor rebalance the portfolio. I calculate the portfolio weights and graphically show how they are distributed. And similar to what I did for the in-sample testing, I also examine the portfolio performance under market stress by calculating each portfolio Sharpe ratio for the 2.5% worst investment returns.

I assume that the investor does not receive any dividends from holding a long position on the assets.

4. Portfolio Optimization

4.1. In-Sample

4.1.1. Minimum variance portfolio

While the linear regression results clearly demonstrate the superior hedging properties of volatility assets over the other assets, the investor still does not know how all assets perform when part of a portfolio. And more importantly, the investor still does not know whether a portfolio which includes a volatility asset performs better than a portfolio which includes a non-volatility asset.

In this section I form multiple portfolios. Each portfolio consists of the unhedged equity investment, a risky asset and a riskless asset. Each portfolio is formed by minimizing its variance and targeting an annual expected excess return of 5.7%[2]. I calculate the standard deviation, Sharpe ratio and optimal weights of each portfolio – results are summarized on Table 6.

Results show that the Sharpe ratios of the original equity investments are 0.3194 and 0.2336, respectively. When it comes to the US market equity investment, all portfolios have a Sharpe ratio that is higher than the original investment in the S&P 500. And among these, the portfolio which includes VIX is the one with the highest Sharpe ratio – hence this is the preferred portfolio because it not only preserves but also increases the equity investment value. Last but not least, to form this

[2] The target return is simply the observed historical excess return of the S&P 500 or DAX 30.

portfolio an investor is required to invest 38.57% on the S&P 500, 7.11% on the VIX and 54.32% on the riskless asset.

When it comes to the European equity investment the results are similar i.e., all portfolios have a Sharpe ratio that is higher than the original investments in the DAX 30. And similar to the US market investment, adding a measure of volatility to the original investment also improves the Sharpe ratio - the portfolio which includes VDAX is the one with the highest Sharpe ratio. This is then the preferred portfolio as it not only preserves but also increases the equity investment value. To form this portfolio an investor is required to invest 36.19% on the DAX 30, 11.01% on the VDAX and 52.8% on the riskless asset.

The in-sample portfolio optimization results are consistent for both investments and further demonstrate the superior hedging properties of volatility assets. A volatility-based strategy (one that includes a volatility index) decreases the risk associated with an unhedged equity investment from 17.84% to 5.56% and from 24.40% to 7.30% in the case of the US and European investment, respectively. And this is achieved while targeting the same level of return of the unhedged investment. No other portfolio achieves similar risk reduction and the corresponding performance improvement. Furthermore, these volatility-based portfolios only require a small investment on the volatility index to achieve such a superior performance: a 7.1% investment in the US investment and an 11% investment in the European investment.

4.1.2. Portfolio performance under market stress

An investor might also be interested (and perhaps more concerned) about the performance of the different portfolios under extreme market conditions. To address this point, I test 1) the strength of each asset and 2) the performance of each portfolio for the 2.5% worst investment returns. I do that by first regressing the investment excess returns against the assets excess returns for the 2.5% worst investment returns and secondly by calculating each portfolio Sharpe ratio for the 2.5% worst quantile - results are summarized on Table 7 and 8.

When it comes to US investment, the only regression coefficient that is negative and statistically significant is the one for VIX. VIX’s regression coefficient is (-2.5002) and is therefore the only asset that maintains the same hedging properties even under market stress. The negative regression coefficient that Swiss francs showed for the entire sample, it is now positive. In fact, the regression coefficients for all other assets indicate that they offer poor hedging benefits which makes them not suitable to be used as equity hedges.

When it comes to European investment, the regression coefficients are negative and statistically significant for VDAX and gold. VDAX’s regression coefficient is greater (-2.4713) than the regression coefficient of gold (-0.2976) which indicates that the volatility index is a stronger hedge. The coefficient results also show that only these two assets can be used as equity hedges under market stress as all the other assets show either positive or not statistically significant regression coefficients.

In conclusion, regression results show that even under extreme market stress volatility assets maintain their superior hedging strength as equity hedges. Next I calculate the Sharpe ratio of each portfolio in the 2.5% worst quantile – results are presented in Table 9.

The Sharpe ratio results for all the S&P 500-based portfolios are negative indicating that an investor should expect to see a reduction in the value of its investment when markets perform in the 2.5% worst quantile – the negative impact ranges from 258% to 33%. The portfolio with the best Sharpe ratio (i.e., less negative) is the one that includes VIX (-0.3354). First and foremost, this result is better than the one for the equity investment: if left unhedged, the investor would see a 259% reduction in the investment value. Secondly, while this volatility-based portfolio is not 100% effective at protecting the value of an equity investment under extreme market conditions it is nonetheless the best at mitigating the negative impact from market stress as all other portfolios show much worst Sharpe ratio results.

The Sharpe ratio results for all the DAX 30-based portfolios are also negative which also indicates that an investor should expect to see a reduction in the value of its investment when markets perform in the 2.5% worst quantile – in this case the negative impact ranges from 352% to 48%. Here as well the portfolio with the best Sharpe ratio (i.e., less negative) is the one that includes VDAX (-0.4879). And this result is also better than the one for the equity investment: if left unhedged, the investor would see a 375% reduction in the investment value. And similar to the VIX portfolio, while the VDAX portfolio is not 100% effective at protecting the value of an equity investment under extreme market conditions it is the best at mitigating the negative impact from market stress as all other portfolios show much worst Sharpe ratio results. Sharpe ratio results for the 2.5% worst quantile show that the portfolios that include the VIX and the VDAX are the best at mitigating the negative impact arising from market turmoil.

At this point in the study and based on the results of the different statistical tests, it is clear that 1) volatility assets are the best equity hedges and 2) a volatility-based hedging strategy is the most effective at hedging an equity investment. But since these are in-sample results I still need to provide evidence that this hedging strategy actually works and whether a dynamic or a buy & hold approach is the appropriate one. For that purpose, in the next section I form multiple minimum variance portfolios by using the out-of-sample method.

4.2. Out-of-Sample

4.2.1. Minimum variance portfolio

In this section, I form multiple portfolios using an out-of-sample method. As discussed before, the basic idea is to select an initial number of observations from the original historical dataset, add one daily extra observation and recalculate the portfolio Sharpe ratio on a sequential basis. Once more, the objective here is to identify which portfolio has the highest Sharpe ratio. Results are presented on Table 9.

When it comes to the S&P 500-based investment, all portfolios have positive Sharpe ratios. But among these, the preferred portfolio is the one which includes VIX because it has the highest one (0.8840). To form this portfolio an investor is required to invest 42.83% on the S&P 500, 8.14% on the VIX and 49.03% on the riskless asset.

Regarding the DAX 30-based investment, all portfolios have positive Sharpe ratios. And once again, the preferred portfolio is the one that includes a volatility asset (in this case, VDAX) because it has the highest Sharpe ratio (0.9493). To form this portfolio an investor is required to invest 42.53% on the DAX 30, 17.80% on the VDAX and 39.67% on the riskless asset.

The out-of-sample portfolio optimization results are consistent for both investments and, once more, demonstrate the superior hedging properties of volatility assets. A dynamic volatility-based hedging strategy returns a Sharpe ratio that is both greater than the Sharpe ratio of any other portfolio and greater than the Sharpe ratio of the original unhedged investment. The Sharpe ratio for the unhedged US investment is 0.3194 while the Sharpe ratio for the volatility-based portfolio is 0.8840; in the European case, the unhedged investment Sharpe ratio is 0.2336 while the hedged portfolio Sharpe ratio is 0.9493. This improvement in performance is achieved by a significant reduction in risk while targeting the same return level of the unhedged equity investment. Specifically, for the US investment results show a reduction in risk from 17.84% to 6.22% and for the European investment, a reduction from 24.40% to 12.54%.

The out-sample portfolio optimization results are in line with the results seen in the in-sample testing which reinforces the strength and effectiveness of a dynamic hedging strategy. In the in-sample testing both volatility-based portfolios performed better than the other portfolios and the same happens with the out-of-sample testing. Further to this point, there was no guarantee that this would be the case because in the in-sample optimization portfolios are created only once and the corresponding Sharpe ratios calculated only once. On the other hand, in the out-of-sample optimization, both portfolios and Sharpe ratios are re-calculated multiple times (more precisely, 4568 and 3034 times for the US and European investments, respectively) which increases the chances of having very different optimization results. But that was not the case and I don’t observe a significant deterioration of portfolio performance with the out-of-sample testing.

Hence the portfolio optimization results clearly support a dynamic hedging strategy and provide evidence that this strategy is the most effective at hedging an equity investment.

4.2.2. Portfolio performance under market stress

As discussed on section 4.1.2., investors are likely interested in evaluating the hedging strategy performance under extreme market conditions. In this section I test the performance of each portfolio for the 2.5% worst investment returns but now using an out-of-sample approach - results are presented on Table 10.

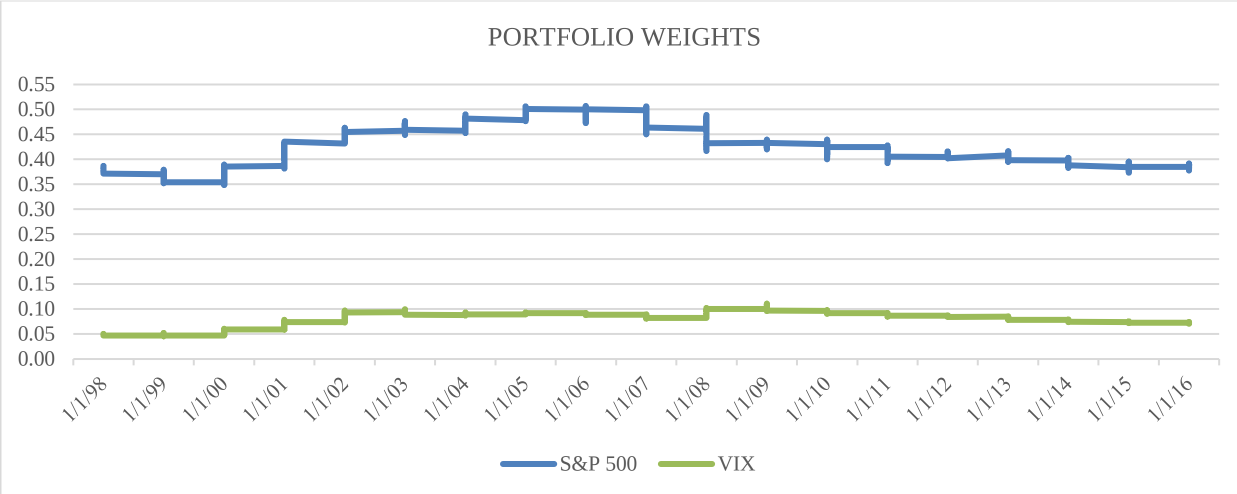
The Sharpe ratio for the VIX portfolio is -4.3132 and for the VDAX portfolio is 7.4201. For both investments, the volatility-based portfolios are the best at protecting an equity investment when returns are in the 2.5% worst quantile. In the case of the VDAX portfolio, Sharpe ratio results indicate that not only this portfolio is the best among all the other portfolios but also that an investor should expect to see an increase in the value of its investment when markets perform in the 2.5% worst quantile. When it comes to the VIX portfolio, although the Sharpe ratio result is negative, it does provide the best protection for the equity investment because the Sharpe ratio of all other portfolios is not better than the VIX portfolio.

In conclusion, the out-of-sample results are consistent for both investments i.e., both portfolios provide the desired protection for an equity investment. Furthermore, results also provide further evidence that a volatility-based portfolios maintain their hedging effectiveness even during market stress.

4.2.3. Portfolio rebalancing

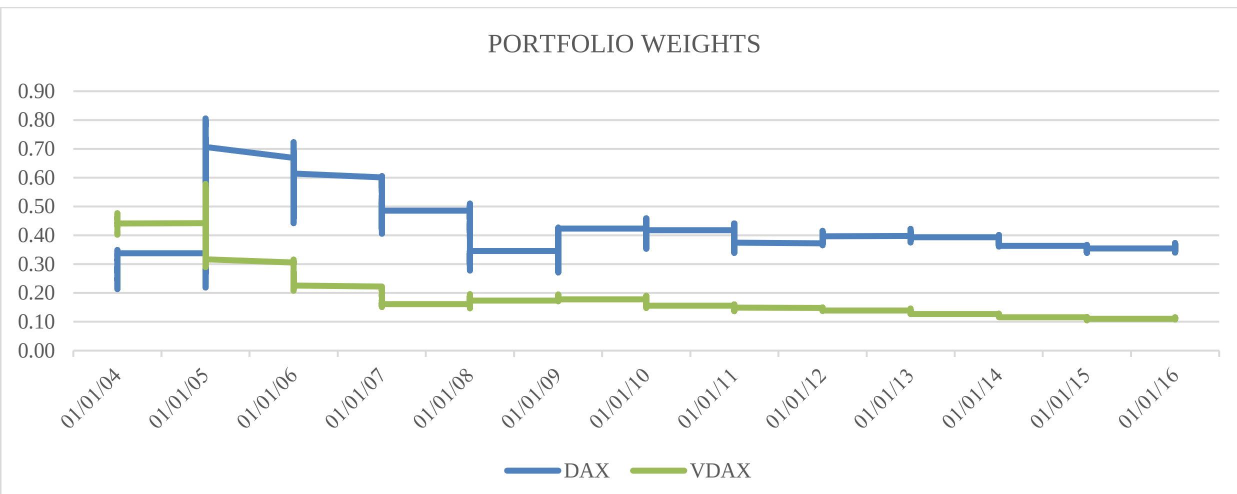
In this section, I investigate how often (if ever) the investor must rebalance its portfolio assuming it adopts a volatility-based dynamic hedging strategy. This point is rather important because of the cost associated with portfolio rebalancing: if rebalancing costs are significant and this hedging strategy requires frequent rebalancing then it might not be economically viable to implement it. Rather than trying to compute a dollar or euro number that reasonably estimates rebalancing costs, I focus instead on how the VIX and VDAX portfolio weights change. If the portfolio weights change materially, then this implies that the portfolio requires frequent rebalancing; if instead the portfolio weights remain relatively stable then this implies that the portfolio does not require frequent rebalancing.

Graph 1 shows how the US volatility-based portfolio weights change as consecutive new portfolios are formed. It is evident that weights don’t change that materially: S&P 500 weights range from 35% to 50% and VIX weights range from 5% to 10%. This means that the investor does not need to rebalance the portfolio as weights tend not to vary significantly.



Graph 1: Portfolio weights for a risky portfolio (S&P 500 + VIX); based on 4568 out-of-sample portfolios

Graph 2 shows how the European volatility-based portfolio weights change as consecutive new portfolios are formed. After some initial variability, the portfolio weights seem to stabilize at around 35% for the DAX 30 and 11% for the VDAX. After some initial variability, the weights stabilize and the portfolio does not require to be rebalanced on a frequent basis.



Graph 2: Portfolio weights for a risky portfolio (DAX 30 + VDAX); based on 3034 out-of-sample portfolios

What this means is that if the investor adopts a dynamic hedging strategy and has to form new daily portfolios, the new daily portfolio weights would not change significantly and the portfolio performance would not be negatively impacted (overall performance is still better than the unhedged equity investment). Hence, a dynamic strategy is perfectly effective as a hedge to an equity investment with the additional benefit of not increasing portfolio management costs related with portfolio rebalancing.

5. Conclusion

I develop a dynamic volatility-based hedging strategy for an equity investment.

The multiple in-sample and out-of-sample test results support this strategy. Both correlation and regression coefficient results show that the volatility assets (VIX and VDAX) are the strongest equity hedges. The in-sample portfolio optimization results show that a strategy based on portfolios that include a volatility asset performs better than a strategy that is based on portfolios formed with conventional assets. Furthermore, the strategy remains effective even under extreme market stress as seen both by the regression coefficient results and by the Sharpe ratio results. The out-of-sample portfolio optimization tests confirm the in-sample results and provide evidence that a dynamic hedging strategy is the most appropriate one since the portfolio weights don't change significantly over time.

One area that requires further investigation is the one related with tracking error associated to funds that try to mimic both the VIX and the VDAX. None of these two volatility indexes are tradable and since the funds that currently try to mimic them have a large tracking error, this diminishes the effectiveness of a volatility-based hedging strategy. Less effective means less protection and increased risk of losing money on the equity investment. One possible way to think about this problem is to ask how much an investor is willing to accept as tracking error vis a vis the protection benefit it receives in implementing this strategy. And related to that how much is an investor, with a certain risk profile, willing to pay a portfolio manager in order to have a certain guaranteed expected return for a certain (lower) risk level.

One other area that also requires investigation is on how liquidity impacts this strategy. There are not many funds that mimic volatility indexes which makes the market small and potentially illiquid. What type of liquidity premium is associated to these funds and how will that impact the overall profit/loss of this hedging strategy.

Appendix

Table 1: Variables description and data sources

Variable	Description	Data	Source	Abbrev
S&P 500 Index	Standard and Poor's 500 Index is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all major industries.	1990-2016	Bloomberg	SP 500
Deutsche Boerse AG German Stock Index DAX	The German Stock Index is a total return index of 30 selected German blue chip stocks traded on the Frankfurt Stock Exchange. The equities use free float shares in the index calculation.	1999-2016	Bloomberg	DAX 30
Chicago Board Options Exchange CBOE Volatility Index	VIX measures market expectation of near term volatility conveyed by stock index option prices. VIX expresses the 30-day implied volatility generated from S&P 500 traded options and thus VIX represents a consensus view of short-term volatility in the equity market	1990-2016	FRED	VIX
Deutsche Boerse VDAX-NEW Volatility Index	The VDAX-NEW® computes the square root of implied variance across at- & out-of-the-money DAX® options of a given time to expiration using the two nearest sub-indices to the remaining time to expiration of 30 days and is calculated on the basis of eight maturities with a maximum time to expiration of two years.	1999-2016	Bloomberg	VDAX
CHF/USD Exchange Rate	The currency pair shows how many U.S. dollars (the quote currency) are needed to purchase one Swiss franc (the base currency)	1990-2016	Bloomberg	CHF/USD
CHF/EUR Exchange Rate	The currency pair shows how many Euros (the quote currency) are needed to purchase one Swiss franc (the base currency)	1990-2016	Bloomberg	CHF/EUR
West Texas Intermediate	Corresponds to the daily crude oil spot prices of West Texas Intermediate (WTI) - Cushing, Oklahoma, dollars per barrel	1990-2016	FRED	WTI
Brent - Europe	Corresponds to the crude oil spot prices (Brent - Europe), dollars per barrel	1990-2016	FRED	Brent
Copper	Corresponds to the end of LME day copper cash price	1990-2016	Bloomberg	COP
Aluminum	Corresponds to the end of LME day aluminum cash price	1990-2016	Bloomberg	ALU
Platinum	Corresponds to the per Troy ounce spot price for Platinum, in plate or ingot form, with a minimum purity of 99.95%.	1990-2016	Bloomberg	PLA
Gold	Corresponds to the gold fixing price 10:30 A.M. (London time) in London Bullion Market, based in U.S. Dollars	1990-2016	FRED	Gold

Table 2: Assets grouped by categories

Categories	Assets	Description
Currency	CHFUSD and CHFEUR	rate at which one currency will be exchanged for another
Commodities	WTI, Brent, Gold, Platinum, Copper and Aluminum	assets used in commerce and which are interchangeable with other commodities
Volatility	CBOE-VIX and VDAX-NEW	Index which represents the market's expectation of 30-day volatility

Table 3: Mean and standard deviation (annualized returns)

	Mean	Standard Deviation
US investment		
S&P 500	0.0871	0.1784
VIX	0.6587	1.0355
CHF/USD	0.0231	0.1153
Gold	0.1168	0.3960
WTI	0.1089	0.3684
Brent	0.0540	0.1639
Platinum	0.0445	0.2093
Copper	0.0698	0.2695
Aluminum	0.0246	0.2127
German investment		
DAX 30	0.0760	0.2440
VDAX	0.4037	0.8736
CHF/EUR	0.0264	0.0825
Gold	0.1729	0.3950
WTI	0.1699	0.3663
Brent	0.0990	0.1845
Platinum	0.0796	0.2286
Copper	0.1183	0.2713
Aluminum	0.0430	0.2175

Note: based on daily data from January 1990 to December 2016 (US investment); based on daily data from January 1999 to December 2016 (European investment)

Table 4: Correlation between equity investments and assets

	Correlation
S&P 500	
VIX	-0.7052
CHF/USD	-0.0740
Gold	-0.0162
WTI	0.1101
Brent	0.0583
Platinum	0.1192
Copper	0.1966
Aluminum	0.1637
DAX 30	
VDAX	-0.6944
CHF/EUR	-0.1424
Gold	-0.0241
WTI	0.1803
Brent	0.1712
Platinum	0.1314
Copper	0.3492
Aluminum	0.2892

Note: based on daily observations from January 1990 to December 2016 (S&P500) and on daily observations from January 1999 to December 2016 (DAX 30).

Table 5: Hedge assessment

	Coefficients	Standard Error	t Stat	R-Square
S&P 500-based investment				
VIX	-4.0924***	0.0498	-82.1056	0.4972
CHF/USD	-0.0478***	0.0078	-6.1253	0.0054
WTI	0.2444	0.0267	9.1466	0.0121
Brent	0.1204	0.0250	4.8205	0.0033
Gold	-0.0149	0.0111	-1.3350	0.0002
Platinum	0.1398	0.0141	9.9090	0.0142
Copper	0.2970	0.0179	16.5581	0.0386
Aluminum	0.1951	0.0142	13.6989	0.0267
DAX 30-based investment				
VDAX	-2.4860***	0.0383	-64.9249	0.4822
CHF/EUR	-0.0482***	0.0050	-9.9807	0.0202
WTI	0.2918	0.0237	12.3306	0.0325
Brent	0.2570	0.0220	11.6883	0.0293
Gold	-0.0182*	0.0112	-1.6234	0.0005
Platinum	0.1231	0.0138	8.9183	0.0172
Copper	0.3882	0.0155	25.0674	0.1219
Aluminum	0.2578	0.0127	20.3210	0.0836

Note: statistics were prepared based on a sample with 6818 daily returns from January 03, 1990 to December 31, 2016 for the S&P 500 based-investment and on a sample of 4528 daily returns from January 05, 1999 to December 31, 2016 for the DAX 30 based-investment. Negative coefficients indicate that the asset is a hedge against the equity investment. *, **, and *** represent statistical significance at the 10% level, 5% level, and 1% level respectively.

Table 6: In-sample minimum variance portfolios

		Return	Risk	Sharpe	***w1***	***w2***	***w3***
Unhedged investment							
	S&P 500	0.0570	0.1784	0.3194	1.0000		
Portfolios							
	VIX	0.0577	0.0556	1.0384	0.3857	0.0711	0.5432
	CHF/USD	0.0569	0.1778	0.3199	0.9887	-0.1076	0.1189
	Gold	0.0567	0.1587	0.3574	0.8065	0.4339	-0.2404
	WTI	0.0571	0.1556	0.3672	0.7111	0.1935	0.0954
	Brent	0.0570	0.1524	0.3737	0.7081	0.2139	0.0780
	Platinum	0.0566	0.1762	0.3215	0.9704	0.1013	-0.0717
	Copper	0.0568	0.1714	0.3313	0.8745	0.1747	-0.0492
	Aluminum	0.0573	0.1746	0.3281	0.9866	-0.1804	0.1938
Unhedged investment							
	DAX 30	0.0570	0.2440	0.2336	1.0000		
Portfolios							
	VDAX	0.0575	0.0730	0.7868	0.3619	0.1101	0.5280
	CHF/EUR	0.0572	0.2111	0.2710	0.8091	1.3016	-1.1107
	Gold	0.0569	0.1159	0.4912	0.2358	0.5537	0.2105
	WTI	0.0568	0.1397	0.4063	0.2446	0.2958	0.4596
	Brent	0.0568	0.1333	0.4259	0.2215	0.3105	0.4680
	Platinum	0.0568	0.1714	0.3313	0.4298	0.5391	0.0311
	Copper	0.0572	0.1524	0.3754	0.2066	0.4683	0.3251
	Aluminum	0.0571	0.2397	0.2383	0.8984	0.2352	-0.1336

Note: ***w1*** refers to the equity investment weight i.e., how much the investor invests in the S&P 500 and DAX 30; ***w2*** refers to the risky asset weight i.e., how much the investor invests in the risky asset; and ***w3*** refers to the risk-free asset weight i.e., how much the investor invests in the riskless asset.

Table 7: Hedge assessment for the 2.5% worst investment returns

	Coefficients	Standard Error	t Stat	R-square
S&P 500-based investment				
VIX	-2.5002***	0.6137	-4.0739	0.0899
CHF/USD	-0.0068	0.0570	-0.1195	0.0000
Gold	0.0190	0.0961	0.1979	0.0002
WTI	0.8636	0.2562	3.3713	0.0633
Brent	0.7493	0.1882	3.9811	0.0862
Platinum	0.3379	0.1472	2.2961	0.0304
Copper	0.3887	0.1295	3.0017	0.0509
Aluminum	0.2703	0.0951	2.8420	0.0458
DAX 30-based investment				
VDAX	-2.4713***	0.6225	-3.9700	0.1243
CHF/EUR	-0.0550	0.0740	-0.7432	0.0049
Gold	-0.2976**	0.1299	-2.2910	0.0451
WTI	0.4152	0.2851	1.4562	0.0187
Brent	0.4793	0.2496	1.9200	0.0321
Platinum	0.0585	0.1920	0.3047	0.0008
Copper	0.5632	0.1752	3.2145	0.0851
Aluminum	0.1462	0.1126	1.2982	0.0149

Note: statistics were prepared based on a sample with 170 daily returns for the S&P 500 based-investment and on a sample of 113 daily returns for the DAX 30 based-investment. Samples represent the worse 2.5% daily returns of the original sample. Negative coefficients indicate that the asset is a hedge against the equity investment. *, **, and *** represent statistical significance at the 10% level, 5% level, and 1% level respectively.

Table 8: In-sample Sharpe ratio for the 2.5% worst quantile

		2.5% worst quantile
S&P 500 (Unhedged)		-2.5902
Portfolios		
	VIX	-0.3554
	CHF/USD	-2.2223
	Gold	-1.7821
	WTI	-1.9819
	Brent	-2.0249
	Platinum	-2.2048
	Copper	-2.4458
	Aluminum	-2.5893
DAX 30 (unhedged)		-3.7549
Portfolios		
	VDAX	-0.4879
	CHF/EUR	-1.2473
	Gold	-1.1050
	WTI	-1.6279
	Brent	-1.5751
	Platinum	-1.6090
	Copper	-1.7590
	Aluminum	-3.5253

Table 9: Out-of-sample minimum variance portfolio

	Return	Risk	Sharpe	***w1***	***w2***	***w3***
S&P 500 based portfolios						
VIX	0.0550	0.0622	0.8840	0.4283	0.0814	0.4903
CHF/USD	0.0276	0.2094	0.1318	0.9155	-0.4615	0.5460
Gold	0.0348	0.2566	0.1355	0.7138	0.1150	0.1712
WTI	0.0501	0.1942	0.2579	0.6605	0.2236	0.1159
Brent	0.0463	0.1848	0.2506	0.6594	0.2345	0.1061
Platinum	0.0551	0.2852	0.1932	0.8874	0.2425	-0.1299
Copper	0.0524	0.2733	0.1916	0.8204	0.1526	0.0270
Aluminum	0.0365	0.2486	0.1470	1.0265	-0.2205	0.1940
DAX 30 based portfolios						
VDAX	0.1190	0.1254	0.9493	0.4253	0.1780	0.3967
CHF/EUR	0.1111	0.4424	0.2512	0.6591	-0.7694	1.1103
Gold	0.0681	0.1127	0.6041	0.0912	0.5714	0.3374
WTI	0.0247	0.1127	0.2193	0.0644	0.2514	0.6842
Brent	0.0214	0.1019	0.2097	0.0606	0.2479	0.6915
Platinum	0.0127	0.1127	0.1127	0.0930	0.4576	0.4494
Copper	0.0286	0.1151	0.2481	-0.0069	0.3917	0.6152
Aluminum	0.0692	0.3610	0.1917	0.2356	0.8573	-0.0929

Table 10: Out of sample Sharpe ratio for the 2.5% worst quantile

	2.5% worst quantile
S&P 500 based portfolios	
VIX	-4.3132
CHF/USD	-28.1444
Gold	-15.5233
WTI	-24.4187
Brent	-26.8431
Platinum	-19.2183
Copper	19.8136
Aluminum	-21.0952
DAX 30 based portfolios	
VDAX	7.4201
CHF/EUR	-6.6097
Gold	-2.3971
WTI	-15.2982
Brent	-17.7672
Platinum	-10.2795
Copper	-17.2651
Aluminum	-7.1907

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