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Forecasting And Stress-Testing The Risk-Based Capital Requirements For Revolving Retail Exposures

Kwamie Dunbar
Sacred Heart University

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Kwamie Dunbar[1]

[1] Assistant Professor of Finance and Economics in the Department of Economics and Finance Sacred Heart University, 5151 Park Ave, Fairfield, Connecticut 06825-1000. Tel: 1.203.396.8068, e-mail: dunbark@sacredheart.edu

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ABSTRACT

This paper presents a tractable and empirically sound technique for generating stressed probabilities of default (PDs) which are then used to derive loss rates for the provisioning of a bank's risk-based capital. This work is in response to the recent regulatory findings attributed to the Supervisory Capital Assessment Program (SCAP) stress tests of 2009 which revealed weaknesses in the existing regulatory and economic capital approaches. The SCAP projected losses of approximately $82.4 Billion in banks' credit card portfolios for 2010, highlighting the need for better forecasting and stress testing of revolving retail exposures.

This study proposes a timely model that will improve the ability of banks to determine the capital adequacy of revolving retail exposures. Using options theory we discuss why an obligor may default and produce estimates of expected losses from our stressed PDs so as to determine loss provisions. This method relies on the simulation of PD distributions via changes in selected macroeconomic variables and the card holder's debt to income ratio (DTIR). The methodology offers the flexibility of being tractable and scalable to data in the issuer's credit card portfolio by geography and credit quality of the obligor.

JEL Classification: **C51, C32, C53, E37, G21**

Operational Risk, Loss Distribution, Stress-test, Consumer Credit Portfolio, Credit Risk, Macroeconomic Risk Factors, Unexpected loss, Capital provisioning, Value-at-Risk

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1.0 Introduction

This paper develops a tractable and empirically sound stress testing procedure for banks' credit card portfolios that generates sound estimates of default probabilities (PDs), which can be stressed under various macroeconomic conditions, see Figure 1. These "*stressed* PDs" can then be used to determine risk-based capital or the portfolio's loss distribution needed to determine the value-at-risk (VaR)[2]. The Federal Reserve's bank stress test results,[3] released on May 7th 2009, suggested that the 19 largest banks within the United States should expect nearly $82.4 billion in credit card losses by the end of 2010, under what federal regulators called the most severe financial crisis since the great depression. Prior to and after this report, a number of credit card issuers' reported significant losses in their credit card portfolios as consumers were forced to make difficult choices on debt priorities given the global economic down-turn brought on by the 2007/2008 credit crisis.

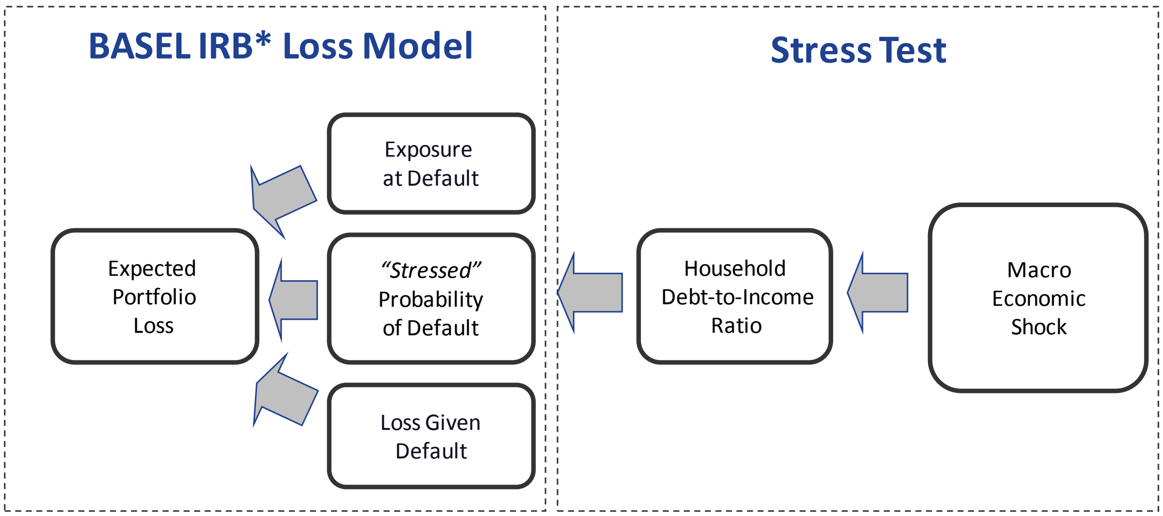
The need to perform accurate stress-tests[4] on the revolving retail exposure of issuers has become increasingly important in evaluating the effects of credit risk to banks. However Sorge (2004) argues that both the accuracy of banks' stress-testing and capital provisioning procedures for unexpected losses on credit card portfolios is usually under-estimated, particularly during periods with adverse macroeconomic shocks. This problem is compounded by the fact that unemployment which was a reliable bell-weather of consumer credit risk (in a number of credit scoring models), appears to be

[2] A discussion of the Vasicek loss distribution follows and can also be found in Vasicek (1987).
[3] To assess the capital positions of the largest U.S. banking organizations, the federal supervisory agencies carried out the Supervisory Capital Assessment Program (SCAP) stress tests in the spring of 2009 (see Board of Governors 2009).
[4] The BIS committee on the global financial system (BCGFS) (2009) defines 'Stress-testing' as – "the techniques used by financial firms to gauge their potential vulnerability to exceptional but plausible events".

decoupling from U.S. credit card losses; for the first quarter of 2011 card losses have been on the decline but the unemployment rate is still at historic highs of around 9.2%.

Figure: 1

Stress-testing model for consumer credit card portfolio: The model generates "Stressed PDs" which are then used in the standard Basel loss framework to determine loss rates.



Debt-to-Income-Ratio captures the factors that are under the internal control of the borrower and driven by a number of latent behavioral variables. The obligor's decision to default is also subject to the Macroeconomy which introduces risk factors. The box "stress test" is the module used to generate the "*stressed PDs*". Debt-to-income is proxied by unemployment rate which provides the flexibility of segmentation by credit quality or geographic region.
*IRB – Internal Ratings Based Approach: refers to a set of credit risk measurement techniques proposed under Basel II capital adequacy rules for banking institutions under which banks are allowed to develop their own empirical model to quantify required capital for credit risk. Banks can use this approach only subject to approval from their local regulators.

Conventional studies have considered tail events of historical episodes to devise scenarios in order to obtain a more precise estimation of bank credit risk when the market is stressed.[5] Bank regulators have also required banks to estimate this risk-based capital at a very high level of confidence so as to avoid any unexpected losses due to the invalid assumption on normality.[6] In response to this, some banks have taken a level of confidence at 99.99% for the computation of VaR, representing a probability of 0.01% for banks to experience such severe credit losses under stressful economic

[5] See details in Froyland and Larsen (2002), Hoggarth and Whitley (2003), Mawdsley *et al*. (2004) and Bunn *et al*. (2005). In fact, some studies have taken into account probabilistic elements and have explicitly considered the correlation among macroeconomic variables and default rates (Wilson (1997a, 1997b), Boss (2002), Virolainen (2004), and Gasha *et al*. (2004)).

[6] Under the Basel II requirement, a higher level of confidence (such as 99.99%) is necessary to compute the required capital amount. See Hugh *et al*. (2005) for more interpretations.

conditions. While this probability may appear low, it can still be under-estimated if the normality assumption is too strong for the default rate. More importantly, the preceding arguments clearly show the need for proper estimation of the probability distribution of portfolio losses needed to determine the default rate in either normal or adverse conditions.

The depth and length of the 2008/2009 financial crisis has demonstrated that the risk management models of banks were not able to effectively measure their exposures at risks. Moreover, recognizing that many of these banks suffered trading losses that notably exceeded their minimum capital requirements set by its original 1996 framework during the crisis, the Basel Committee on Banking Supervision (2009) revised its regulatory framework for trading portfolios. Such losses suggest that risk management systems based on the original framework were ineffective in preventing banks from taking substantive tail risk in their trading portfolios. Hence, the Basel committee has introduced a number of new proposals for tackling the challenges presented by the financial crisis. Its recommended proposal to overhaul the existing VaR models with the introduction of a new stressed VaR measure has come under fierce criticism. Particularly since the VaR framework which is often used in risk management tries to illustrate a measure of potential exposure from a complex system of underling variables and assumptions on different elements which is computationally too difficult to estimate, particularly in stress phases (Miele and Elisa (2011)).

Few will dispute that the Basel II framework has flaws, which has been exposed by the financial crisis. The losses sustained by banks on structured credit exposures far outweighed anything their value-at-risk models suggested they could lose. However, while some believe using VAR is incoherent, it does hold some important advantages for bank regulators. For instance, most banks currently use VAR, so changes to the measure itself should present few modeling challenges to banks. The difficult part will be identifying a suitable historical period of market stress. To this concern the Basel Committee says the

12-month period relating to losses in 2007 and 2008 would qualify as a period of stress, although other relevant periods could be considered by banks, subject to supervisory approval. In addition, despite all the criticism, the stressed VAR capital charge should help to make bank capital less pro-cyclical – a key goal of bank regulators since the 2008/2009 market crash, this is an important element that shouldn't be overlooked.

To assess appropriate loss provisions levels we need to understand the shape of the loss distribution for the bank's revolving retail exposure (portfolios of credit card debt). Provisioning excess capital for unexpected losses may then be allocated in proportion to the distance between a specified, far-tail loss rate of the distribution and the expected loss rate. Models applying simulation techniques to generate loss probability distributions in recent years have become standard tools to evaluate loss provisioning for consumer credit risk in the mortgage loan sector (Calem and LaCour-Little (2001)). However, loss provisioning research for credit card portfolios has received less attention. We employ a semi-parametric procedure for modeling our credit loss distributions. The procedure combines empirical survival curves, simulated scenarios of credit card debt charge off rates, and economic growth rates, calibrated to the debt to income ratio to yield probability distributions over losses due to default.

This approach has a number of advantages. First, it takes full advantage of the available historical data on credit card portfolio performance. Second, it employs a theoretical framework that is built on the consumer's debt-to-income (DTIR) experience.[7] Here we assume that the consumer's DTIR is driven by a number of latent behavioral variables (BVs) and which will be a determinant in his decision to default based on some embedded real option of default. Third, it employs a non-parametric re-

[7] There are two types of debt-to-income ratios that lenders compute when households seek credit, These are (a) The front-end ratio, also called the housing ratio, shows the percentage of an obligor's income that goes toward housing expenses, including monthly mortgage payments, real estate taxes, homeowner's insurance and association dues. (b) The back-end ratio shows the portion of an obligor's income is needed to cover all monthly debt obligations. This includes credit card bills, car loans, child support, student loans and any other debt that shows on the credit report, plus mortgage and other housing expenses. Lenders typically say the ideal front-end ratio should be no more than 28 percent, and a back-end ratio not exceeding 36 percent.

sampling method (as in Black and Morgan (1998)) to derive probability distributions over future paths of employment, whereby risk-factor scenarios are generated by Monte-Carlo re-sampling from historical output growth data. Fourthly, the model is scalable and can be adapted to regional data by banks that need to report on regional default experiences to regulators. In this paper, we restrict our attention to the internal rating based (IRB) approach[8] which allows banks to use internal estimates of credit risk to determine loss rates.

The remainder of the study is organized as follows; we first discuss the existing models that predict the risk of default of an obligor – gleaned from a survey of the academic and practitioner's literature. We conclude that consumer risk models can be made more accurate in their predictions of the probability that a borrower will default and more informative about the level of value at risk when states of the economy and dynamic behavior are included, appropriately, in models of consumer risk using a DTIR-PD framework. Section 3 describes the analytical framework which utilizes the two-state Merton-type one factor model employed by the Basel Accord framework for calculating risk weights. The general model framework is augmented with the PD-DTIR methodology. Section 4 presents the data. Section 5 discusses the results of the PD-DTIR model. Based on the estimated coefficients, stress-testing is then done by simulation of different economic conditions. Section 8 gives a summary and concluding remark.

2.0 Related Literature

Despite the rapid growth in credit card debt and its related delinquency rates, surprisingly no work has been done on developing a framework for generating a mixture of "*stressed PDs*" that can be

[8] Several large banks follow the IRB approach required by the Basel capital accord for determining expected loss rates. While some inputs to the IRB approach such as the LGD and EAD maybe directly observable from the historical data, the PD is usually not, and needs to be estimated. The accuracy with which this is done can have significant effects on loss rates calculations.

used in the standard Basel II LGD-EAD-PD loss model for use in determining loss rates in either normal or adverse conditions. This may be partly due to the fact that banks and rating agencies do not generally publish the data they hold on individuals or the results of their credit scoring models. The earlier work of Bridges and Disney (2001) sheds some light on these credit scoring models and lists a number of behavioral variables (BV) typically found in these models of default such as; an individual's monthly income, outstanding debt, financial assets, type of bank account, time in job, whether the individual has ever defaulted on a previous loan and whether they own or rent their homes. Crook and Bellotti (2009) argue that a weakness in this behavioral framework is that the probability of default is based on static behavioral characteristics[9] which were obtained at the time of credit application. Hence variants of these behavioral models have attempted to include predictors that vary over time, such as recent repayment and account activity, but they rarely, if ever, include indicators of the macro economy. There is considerable evidence that the state of the macro economy affects, the chance that an individual will default in the future (Crook and Banasik (2005), Boss (2002), Gasha *et al* (2004), Virolainen (2004) and Whitley *et al.* (2004)) which may also affect the VaR of a portfolio of credit card loans.

The model frameworks that have been proposed for revolving retail exposure can be bifurcated into two main approaches which range from continuous to discrete time survival models (see Crook and Bellotti (2010) for a discussion). Thomas *et al* (2001) describe how to use a Markov chain stochastic process as a dynamic model of delinquency. However, the approach they describe does not allow for model covariates, though the model can be divided into separate consumer segments for the modeling of different risk groups. A few researchers have used survival analysis as a means to build dynamic models since this approach readily allows the inclusion of BVs and macroeconomic variables (MVs) as

[9] Such models that are used to determine whether an applicant should be granted credit are based on data collected at the time of application that then remain fixed. Typically, this is information taken from a completed application form and a credit score for the individual provided by a credit bureau.

time-varying covariates (TVCs). Bellotti and Crook (2009) follow this path using a Cox proportional hazard survival framework to model time to default for a large database of credit cards. They include MVs but not BVs as TVCs and found a modest improvement in predictive performance in comparison to a static logistic regression.

Instead of a dynamic MV or BV model Breedan *et* al (2008) and Breedan and Ingram (2009) develop a default generating framework where the default rate of a portfolio over time is explained as a function of duration time, calendar time and vintage. But they do not present the results of a stress test for their portfolio and it is debatable whether simulating a parameterized function of calendar time is the same as simulating macroeconomic variables that are related to the probability of default at the consumer account level. Rösch and Scheule (2004) assume a Merton one factor model and estimate loss distributions for credit cards, mortgages and other consumer loans in the US. But they use aggregate default rate data and omit variables specific to the obligor. It is also unclear how they preserved the correlation structure between the MVs in their model. Boss (2002) uses a similar dynamic model structure for simulation-based stress tests on corporate loans.

A common factor in all these studies in the academic and practitioner's literature is the fact that both BVs and MVs are useful explanatory covariates for revolving retail exposure. However none have reported using these models for stress-testing, and as illustrated by the 2009 SCAP stress tests, financial institutions and regulators are more interested in consumer credit risk models for estimating future losses in normal and adverse economic conditions. Vasicek (1987) suggests that the amount of capital necessary to support a portfolio of retail debt securities depends on the probability distribution of the portfolio loss, which is also important in calculating of VaR. Hence, deriving scenarios of "*stressed PDs*" should be very useful to the standard LGD-EAD-PD loss model.

In contrast to Rösch and Scheule (2004), we propose a model framework in which an obligor's PD is driven by a factor that is internal and specific to each obligor's economic condition and which is itself responsive to factors outside the obligor's control; the consumer's PD-DTIR (see Figure 2). This framework allows for the inclusion of both BVs and MVs as significant predictors in the default process. The proposed model suggests that the PD is predicated on the agent's DTIR distribution, where that DTIR ratio acts strategically to indicate an individual's debt position and month to month ability to avoid default. A panel study by Black and Morgan (1998) found that the change in the household debt burden, measured as the ratio of debt payments to income, was the main determinant of default risk. If a household experienced income losses of 1 percentage point above the average level, it was 9.7% more likely to be in payment delinquency and eventual default. Not surprisingly, but as we will make use of later, they also found a positive and significant relationship between income levels and default risk> This is supported by the work of Gross and Souleles (2002) which found that the unemployment rate had a statistically significant effect on bankruptcy.

Like Rösch and Scheule (2004) our PD-DTIR model framework uses logistic regression analysis to predict the default probabilities. Since credit data, and in particular monthly account records are discrete this model is superior to continuous time survival analysis. This discrete survival approach also has the advantage of being more computationally advantageous since probability forecasts involve simple summations over time periods, rather than mathematical integration which may be complex when TVCs are included in the model. Discrete survival models have been applied successfully in the analysis of personal bankruptcy and delinquency (Gross and Souleles (2002)), mortgage terminations (Calhoun and Deng (2002)) and in default risk and foreclosure in the US subprime market (Gerardi *et al* (2008)).

3.0 The Analytical Model

Similar to Wilson (1997a, 1997b), Boss (2002), Virolainen (2004) and Wong et al. (2006), the framework for stress testing banks' revolving retail exposure to macroeconomic shocks comprises: (i) an empirical model with a system of equations describing credit risk and macroeconomic dynamics, and (ii) a Monte Carlo simulation for generating distribution of possible default probabilities and credit losses.

The model we use is a variant of the individual two-state-one-factor Credit Metrics model employed in the Basel II framework for calculating risk weights. The two states are referred to as "default" and "non-default." The discrete-time process for the normalized return R_{it} on the assets of borrower i at time t is assumed to follow a one factor model of the form

$$R_{it} = bF_t + \sqrt{1-b^2 U_{it}} \tag{1}$$

where

$$F_t \sim N(0,1) \;\; U_{it} \sim N(0,1)$$

$(i=1,...,N_t.\ t=1,...,T)$ are normally distributed with mean (θ) zero and standard deviation (σ) one. Idiosyncratic shocks U_{it} are assumed to be independent from the systematic factor F_t and independent for different borrowers. All random variables are serially independent.

The exposure to the common factor is denoted by b. Under these assumptions the correlation (ρ) between the normalized asset returns of any two borrowers is b^2. We will refer to this correlation as asset correlation. As in the Basel Accord we assume that borrowers can be grouped into homogenous segments. In each segment a borrower defaults at a time t if the return on his asset falls short of some threshold β_0, i.e.,

$$R_{it} < \beta_0 \Leftrightarrow Y_{it} = 1$$

$\left(i=1,\ldots,N_t.\;\; t=1,\ldots,T\right)$, where Y_{it} is an indicator variable with

$$Y_{it}=\begin{cases}1 \text{ borrower } i \text{ defaults at } t\\ 0 \text{ else}\end{cases}$$

and where Y_{it} maybe considered an embedded real option for borrower i at time t. Here, the borrower has an option to default and will exercise this option based on changes in his income and debt levels. The probability of default is based on the borrower's debt to income ratio with the option to default being exercised when the borrower no longer has the ability or perceived means to repay debt, where this option to default maybe represented as[10];

$$\mathrm{Y}_{it} = \lambda_{it} = \max\left(0, (D_{it} - A_{it})\right) \tag{2}$$

Unlike the earlier approaches discussed in section 2, the DTIR approach allows us to capture the effects of the BVs without having to explicitly identify all possible BVs that affect the borrower's probability to default. D_{it} represents the i^{th} borrower's debt and A_{it} represents income. When D_{it} exceeds A_{it} there is an incentive by the borrower to exercise his embedded default option.[11] Another important fact regarding equation (2) is that $(D_{it} - A_{it})$ reflects the borrower's ability to generate either present or future income streams sufficient to satisfy current and future debt obligations.[12]

Since income levels (and local unemployment rates) have a positive and significant effect on default rates (Black and Morgan (1998) and Gross and Souleles (2002)), we use changes in the unemployment rate over the prior year (ΔUE_t) to proxy the DTIR. This approach offers two major

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[10] Note that the customer's option to default for cash flow or leverage reasons is distinctly a non-linear relationship. These relationships are handled in a probabilistic manner, which captures the economic idea that after a certain threshold level, a customer's payment practices will become very sensitive to macroeconomic shocks.

[11] This is consistent with Merton (1974) who suggests that a borrower defaults when the value of his assets falls below a certain threshold level.

[12] We use the narrower definition of household assets; income, because based on the available Federal Reserve data, U.S. households' savings rate was less than 2% of disposable income for several years and the available assets needed by the median family to insulate a loss of labor income was very small (well below the advisable 6 months of savings).

advantages. Firstly, rising unemployment signals loss incomes and a possible increase in the obligor's "*right*" to exercise his real option to default.[13] Secondly, the use of unemployment allows for segmentation of the credit risk model by geographic region. Within the United States the Bureau of Labor Statistics (BLS) provides unemployment data at both the national and regional levels, whilst the States also track their local unemployment. This affords us much more flexibility than the Federal Reserve's aggregate debt to disposal income data.[14] This is of utmost importance to banks that have been required by the Federal Reserve to segment stress-test models by geographic region within the United States. A number of business publications have recently pointed to the decoupling of unemployment as a noteworthy bell-weather of credit card losses. However as discussed in section 5, changes in the unemployment rate from the prior period is a better predictor of consumer credit risk.

The probability of default at time t for borrower i within a given segment is then

$$\begin{aligned}\lambda &= P\left(Y_{it}=1\right)=P\left(R_{it}<\beta_0\right)\\ &= P\left(bF_t+\sqrt{1-b^2U_{it}}<\beta_0\right)=\Phi\left(\beta_0\right)\end{aligned} \tag{3}$$

where $\Phi(\bullet)$ denotes the cumulative standard normal distribution function. This probability is actually a conditional probability, given the borrower has survived until time t. Conditional on a realization f_t of the common random factor at time t the default probability becomes

$$\lambda\left(f_t\right)=P\left(U_{it}<\frac{\beta_0-bf_t}{\sqrt{1-b^2}}\right)=\Phi\left(\frac{\beta_0-bf_t}{\sqrt{1-b^2}}\right) \tag{4}$$

The conditional default probability can also be expresed in terms of the unconditional probability of default and the asset correlation:

[13] The aggregate debt-to-income data of the Federal Reserve is not reflective of the household budgetary realities of the median family. Plus it does not allow for segmentation by geographic region.
[14] Because the national credit bureaus do not maintain real-time data on obligors' incomes similar to the extensive data on debt service commitments, researchers are forced to find representative proxies for household's income.

$$\lambda(f_t) = \Phi\left(\frac{\Phi^{-1}(\lambda) - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right) \quad (5)$$

where $\Phi^{-1}(\bullet)$ denotes the inverse cummulative standard normal distribution function. As described in Finger (1998), the realization f_t of the common random factor can be interpreted as the state of the economy in t. The conditional default probabilities decrease in good years (positive factor realization) and increases in bad years (negative factor realization). Conditional on the realization of the common random factor defaults are independent between borrowers and the number of defaults $D(f_t)$ at time t for a given number N_t of borrowers is (conditional) binomially distributed with probability $\lambda(f_t)$, i.e.,

$$D(f_t) \sim B(N_t, \lambda(f_t))$$

where $B(\bullet)$ denotes the binomial distribution (see Gordy and Heitfield (2000)).

Expression 3 assumes that there is a default threshold which is time invariant and that the unconditional default probability is constant over the time period under consideration. A more advanced specification is to model time-varying default probabilities and explicitly take their fluctuation during the business cycle into account. This is done by including observable risk factors, i.e. macroeconomic (MV) risk factors. Let $z_t = (z_1,...,z_k)'$ denote a $K-$vector of MV risk factors and $\beta = (\beta_1,...\beta_n)'$ the vector of sensitivities with regards to these factors. Then within any given consumer credit card segment[15] the probability of default conditional on the observable risk factors is

$$\lambda(z_t) = P\left(bF_t + \sqrt{1-b^2 U_{it}} < \beta_0 + \beta' z_t\right) = \Phi(\beta_0 + \beta' z_t) \quad (6)$$

[15] Here segmented may be considered in terms of the bank's geographic distribution of credit card issuance or it could be in terms of the credit quality of the cardholders. Banks typically categorize credit card holders based on credit riskiness into one of several internally determined credit categories.

Thus the default probability depends on the state of the economy which is represented by the variables in the vector z_t . A positive sensitivity with respects to the risk factor leads to a higher default probability and vice versa. Again conditioning on a realization f_t of some common random factor the probability loss distribution is

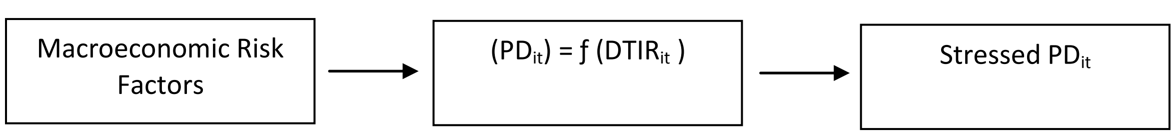
$$\lambda\left(f_t, z_t\right) = \Phi\left(\frac{\Phi^{-1}\left(\lambda(z_t)\right) - \sqrt{\rho f_t}}{\sqrt{1-\rho}}\right) \tag{7}$$

In this model the parameters can be estimated without the observation of asset returns. Only defaults have to be observed as dependent variables. The asset returns can then be treated as latent variables. This is especially convenient for retail credit risk where asset returns cannot be observed (see discussion in (Vasicek (1987) and Rösch and Scheule (2004)). For a given time series of defaults and macro economic variables the parameters β', β_0 and the asset correlation in models (3) and (6) can be estimated by maximum likelihood using a threshold model as in Gordy and Heitfield (2002). However we use a logit model to transform the time series of default rates to λ_i, see Appendix A.

The MV risk factors used in expressions (6) and (7) is the time t changes in the real GDP over the prior period, which provides the parameters β_0 and $\beta' z_t$ respectively. The tractability of this dynamic framework allows us to both predict the $n-\text{step}$ ahead default forecast and to stress the PD under different macro economic conditions. See Figure 2 below;

Figure: 2

Derivation of Stressed Probabilities of Default for Forecasting Future Portfolio Loss rates



UE is used as a proxy of the obligor's DTIR. The obligor's DTIR is a function of the macroeconomic risk factors which results in a mixture of PDs that can be used to determine loss rates in various economic scenarios. Box 2 involves a Logit tansform of the observed loss rates to PDs conditioned on DTIR.

In the economic literature, growth slowdowns typically coincide with rising unemployment. This negative correlation between real GDP growth and the unemployment rate has been named “Okun’s law”, after Arthur Okun (1962) demonstrated that changes in the unemployment rate from one quarter to the next moved with quarterly growth in real output. Following Knotek (2007) we use a *dynamic* representation[16] of Okun’s law in (8) to captures the contemporaneous relationship between real output growth and movements in the unemployment rate.

$$\Delta UE_t = \alpha_0 + \alpha_1 g_t + \alpha_2 g_{t-1} + \alpha_3 g_{t-2} + \alpha_4 \Delta_1 UE_{t-1} + \alpha_5 \Delta_2 UE_{t-2} + \varepsilon_t \quad (8)$$

where a common form for the dynamic version reflects current real output growth (g_t), past real output growth $(g_{t-1,} g_{t-2})$, and past changes in the unemployment rate as variables (UE_{t-1}, UE_{t-2}) on the right side of the equation. These variables explain the current change in the unemployment rate on the left side. Utilizing Monte Carlo simulations to provide possible paths of the macroeconomic risk factors (assuming that LGD and EAD are directly observable from the portfolio), the model produces distributions of PDs and Losses by macroeconomic scenario.

3.1 Generating the Loss Distributions

Since the PDs and the correlations can be estimated for each class of credit card debt exposures[17], we first derive the loss distributions separately for each exposure class. We then agggregate the marginal distributions for the different credit segments into a single portfolio

[16] *The dynamic version* suggests that both past and current output can impact the current level of unemployment. In the difference version of Okun’s law, this implies that some relevant variables have been omitted from the right side of the equation. Secondly, including past changes in the unemployment rate as variables on the right side eliminates serial correlation in the error terms.

[17] As discussed earlier, the credit card portfolio maybe segmented in terms of the bank’s geographic distribution of credit card issuance or in terms of the credit quality of the cardholders. Banks typically categorize credit card holders based on credit riskiness into one of several credit categories.

distribution for the firm, see Appendix B. In both steps we compute the expected loss, Value at Risk and the unexpected loss of the time varying default probabilities and estimated correlations.

3.2 The Marginal loss forecasts for the credit card portfolio

Given the parameters of the model, the default distribution of the potential numbers of defaulting cardholders for the $T+1$ period can be estimated as demonstrated in Vasicek (1987). If expression (3) with constant default probabilities is used the probability distribution for the D_{T+1} of defaulting obligor within a risk segment given the number N_{T+1} of obligors in the segment at the begining of the period is

$$PD_{T+!} = \begin{cases} \binom{N_{T+1}}{D_{T+1}} \int_{-\infty}^{+\infty} \begin{bmatrix} \lambda\left(f_{T+1}\right)^{D_{T+1}} \times \\ \left[1-\lambda\left(f_{T+1}\right)^{N_{T+1}-D_{T+1}}\right] \end{bmatrix} \varphi\left(f_{T+1}\right) df_{T+1} & D_{T+1} = 0,1,2,\ldots,N_{T+1} \\ 0 & \text{else} \end{cases} \tag{9}$$

where $\lambda\left(f_{T+1}\right)$ is defined anologously to (4). This distribution depends on the point of the credit cycle only by N_{T+1} since the distribution of the random factor is standard normal at each point of time. The cyclical variations is captured by the credit quality correlations and introduces some uncertainty and skewness into the distribution. If expression (6) is assumed then the probability distribution is

$$PD_{T+!} = \begin{cases} \binom{N_{T+1}}{D_{T+1}} \int_{-\infty}^{+\infty} \begin{bmatrix} \lambda\left(f_{T+1}, z_{T+1}\right)^{D_{T-1}} \times \\ \begin{bmatrix} -1 \\ 1-\lambda\left(f_{T+1}, z_{T+1}\right)^{N_{T-1}-D_{T-1}} \end{bmatrix} \end{bmatrix} \varphi\left(f_{T+1}\right) df_{T+1} & D_{T+1} = 0,1,2,\ldots,N_{T+1} \\ 0 & \text{else} \end{cases} \tag{10}$$

where $\lambda\left(f_{T+1}, z_{T+1}\right)$ is defined analagously to (7). The distribution in (10) explicitly depends on the state of the economy by the macroeconomic risk factors illustrated in (8).

4.0 The Data

4.1 Credit card data

For the empirical evaluation of the model we use the annual charge-off rates filed by commercial banks to the Federal Financial Institutions Council as an approximation of the default rate for a given year. For these analyses we assume this data is for accounts that are in default or has gone three consecutive months delinquent on payments. This definition of default is common in the industry and consistent with the Basel II convention of 90 days delinquency for consumer credit (BCBS (2006)). The proprietary data we seek for these analyses is commercially sensitive and difficult to obtain so we opt for those available at the Federal Reserve's bank. Table 1 presents descriptive statistics on these reported default rates. To evaluate the model's forecasts, an observation date of 1989 to 2009 is set. Since the data runs to the end of 2010, this provides 1 year of test data.

4.2 Historic US macroeconomic data

We also collect data on changes in the unemployment rate over the prior period and growth in real GDP from the Federal Reserve Bank of St. Louis. These are also listed in Table 1. For stress testing, historical values of MVs are taken from 1989 to 2010.

Table: 1

Descriptive Statistics for Macroeconomic Variables (MVs) 1989-2010.

MV	Description	Scource	Descriptive Statistics					
			Min	Mean	Std Dev	Max	Skew	Kurtosis
UE	*Unemployment rate*	Federal Reserve Bank of St Louis*	4.000	6.303	1.624	9.700	0.800	-0.110
RGDP	*Growth in Real Gross Domestic Product*	Federal Reserve Bank of St Louis*	-2.633	2.689	2.044	7.100	-0.810	1.220
Cr	Charge-off Rate	Quarterly Consolidated Reports of Condition and Income*	3.100	5.088	1.674	9.403	1.538	1.997

** The macroeconomic variables (MV) change in %(UE) and growth in Gross Domestic in %(GDP)*
*** Report filed by all U.S. commercial Banks to the Fedreal Financial Institutions Examination Council - www.ffiec.gov*
All Data is monthly and may be seasonally adjusted.

5.0 Discussion of the Results

5.1 Model and coefficient estimates

Expression (3) assumes constant default probabilities over time. In this case, cyclical patterns are attributed to the correlations arising from the geographical or credit quality differences. For these analyses we assume that default probabilities change over time because of the business cycle which is explained by the model's macroeconomic risk factors illustrated in Figure 1. Hence, the exposures of the default probabilities to the risk factors and the random factor are estimated by equation (6). Table 2 presents the estimation results of equation (6). The estimated parameters in Table 2 are then used to forecast the default probability for 2010. These PDs can be stressed to generate PDs under both normal and severe unemployment (DTIR) conditions. The Basel II IRB guidelines suggest that;

$$\text{Expected Losses} = \text{EAD} \times \text{LGD} \times \text{PD}$$

For simplicity, assuming that EAD and LGD equals 1, we then plot the estimated and the observed loss rates in Figure 3, and a forecast of the 2010 expected loss rate.[18]

If the parameter estimates in Table 2 show a positive sign, the default probability increases with the respective variable and vice versa. For instance, in the case of the percentage growth in the 1 year lagged unemployment rate $\left(\Delta UE_{t-1}\right)$, a positive sign on the parameter estimate indicates that an increase of unemployment rate this year leads to a higher probability of default in the next year. The economic plausibility of the other variables can be assessed in a similar way.

[18] The logit default probabilities are obtained from Table 2 by $E\left(C_r \left| \Delta UE_t \right.\right) = \lambda_t = \dfrac{e^{\left(\alpha_0 + \alpha_1 \times \Delta UE_t + \alpha_2 \times \Delta UE_{t-1}\right)}}{1 + e^{\left(\alpha_0 + \alpha_1 \times \Delta UE_t + \alpha_2 \times \Delta UE_{t-1}\right)}}$

Table: 2

Parameter Estimates for the Logistic Regression Model for 1989-2010.

Logit Transform of $C_{rt} = \alpha_0 + \alpha_1 \Delta UE_t + \alpha_2 \Delta UE_{t-1} + \varepsilon_t$

Parameters	Variables	Estimates	Standard Error
α_0	*Constant*	-3.028***	0.041
α_1	*UE*	0.135***	0.046
α_2	*Ue_{t-1}*	0.172***	0.046
R^2	0.678		

** indicates significance at the 90% level of confidence, ** indicates significance at the 95% levelof confidence and *** indicates a 99% level of confidence.*

The results in Table 2 indicate that changes in the current unemployment rate and the lagged 1 period change are significant indicators of direct economic distress on individuals. In particular, obligors who become or remain unemployed will find it more difficult to repay debt. Conversely, if unemployment decreases, then we would generally expect unemployed obligors to find jobs, therefore making it easier for them to repay.

Exhibit 3

Real and Fitted Default Rates - Credit Card Loans - 1989-2010

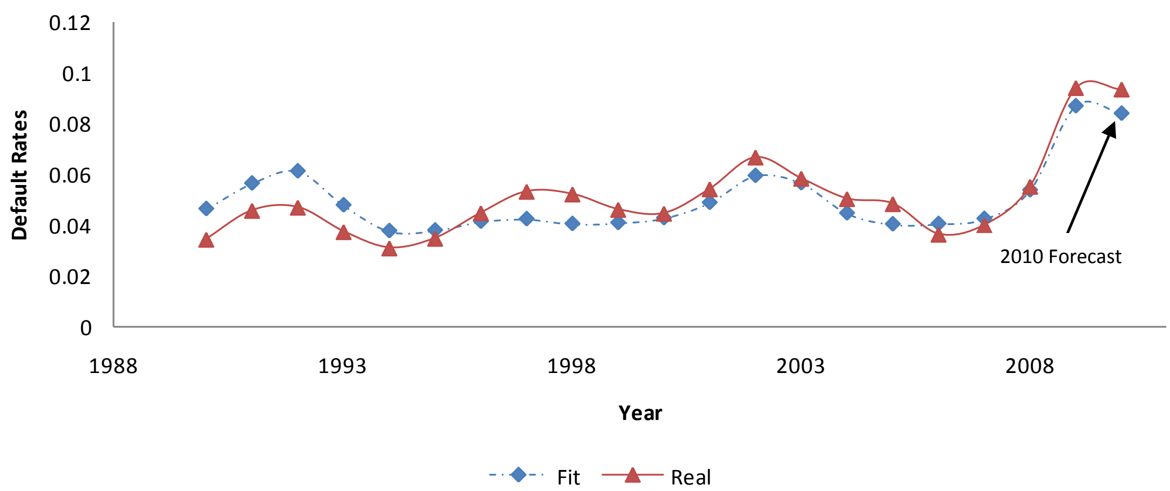


Table 3 presents the statistically significant coefficient estimates for expression 8; several MVs were found to be statistically significant explanatory variables. As in the case of Table 2, if the parameter estimates in Table 3 show a positive sign, then changes in the unemployment rate increases with the respective variable and vice versa. For instance, in the case of the percentage growth in the 1 year lagged real gross domestic product (GDP), a negative sign on the parameter estimate indicates that a decrease of real GDP growth leads to a increased growth in unemployment in the next year. The significance of the MVs in Table 3 allows us to stress the PDs derived from Table 2 to determine loss rates and capital provisioning. For revolving retail exposure of different credit classes, the results in Tables 1 through 3 are replicated for each credit class. These results can then be aggregated using Appendix B to generate a single loss distribution.

Table: 3

Parameter Estimates for OLS Regression Model of MV Risk Factors for 1989-2010.

$$\Delta UE_t = \alpha_0 + \alpha_1 g_t + \alpha_2 g_{t-1} + \alpha_3 \Delta_1 UE_{t-1} + \varepsilon_t$$

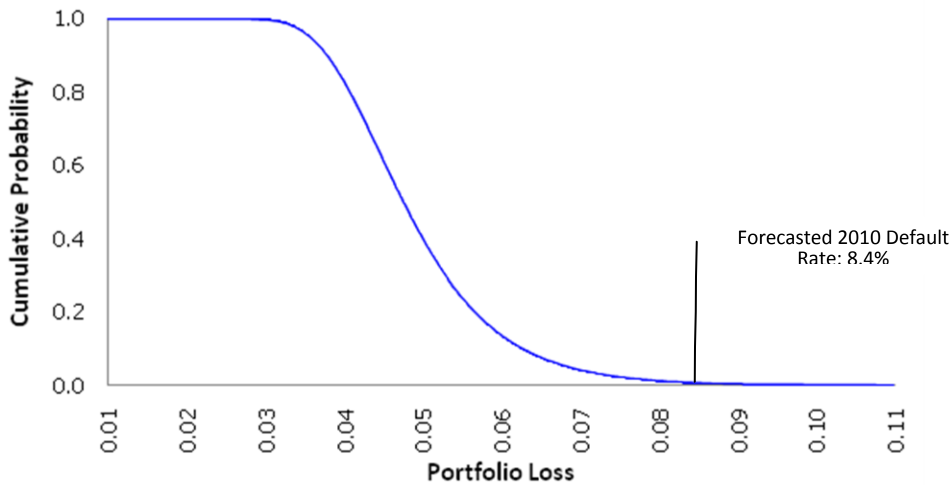
Parameters	Variables	Estimates	Standard Error
α	*Constant*	1.863***	0.399
α_1	*Current Real Output Growth*	-0.418***	0.044
α_2	*Prior period's Real Output Growth*	-0.107*	0.044
α_4	*Past 1 period's Changes in the UE Rate*	0.946***	0.059
R^2	0.944		

** indicates significance at the 90% level of confidence, ** indicates significance at the 95% levelof confidence and *** indicates a 99% level of confidence.*

5.2 Simulated Loss Distribution: Expected and Unexpected Losses

Computer simulations using the estimated PDs reported in Table 2 show that expression (7) appears to provide a reasonably good fit to the tail loss distribution. A stable loss distribution was generated after n = 10,000 Monte Carlo simulations and is illustrated in Figure 4 which plots the simulated cumulative distribution function of the loss in 1 year. The right-hand tail shows risk for more adverse conditions at the 99% percentile.

Exhibit 4:
Simulated Loss Distribution for a Credit Card Portfolio



Note the distribution is constructed with 10,000 simulated future paths of the default rates.

Table: 4
The Value-at-Risk Quantiles of Forecasted Loss Distributions for U.S. Revolving Retail Exposures: For **2010**.

Exposure Class		**Loss Rates**		**Provisions and Requirements**	
Confidence Level (%)	VaR (Percentile of Loss Distribution)	Expected Losses (EL)	Unexpected Losses (UL)	Provisions	Additional Requirements
90.0%	7.44%	5.62%	1.82%	5.62%	1.82%
95.0%	7.84%	5.62%	2.22%	5.62%	2.22%
99.0%	8.51%	5.62%	2.89%	5.62%	2.89%
99.9%	9.00%	5.62%	3.38%	5.62%	3.38%

The number of borrowers is simulated at 100,000: losses are in % of portfolio value, VaR is defined as a quantile of the loss distribution. Unexpected losses is defines as the difference between the Value-at-risk quantile and the Expected loss.

Conventional studies have considered tail events of historical episodes to devise scenarios in order to obtain a more precise estimation of the bank's credit risk when the market is stressed, so as to avoid an unexpected loss due to the invalid assumption of normality (See Hugh *et al* (2005)). Table 4 presents the Value-at-Risk, expected losses along with the unexpected shortfall at the 90 – 99.9%

probabilities for 2010. From Table 4, if the bank needs to cover average losses incurred in the usual course of business then it should make capital provisions for losses of approximately 5.62%. However, if the institution wants to establish loss provisions to serve as a buffer against potentially severe losses (as required by regulators), then it should keep an additional loan provisions of 3.38% (to be adequately covered at the 99.9% level as dictated by the Basel committee on banking).

6.0 Conclusions

This paper presents a PD-DTIR stress testing model for revolving retail exposures of banks in the United States. The PD-DTIR model is considered because it provides a framework for generating a mixture of "*stressed PDs*" for the standard LGD-EAD-PD Basel loss model. Recent regulatory stress tests in the US consumer credit card industry have highlighted weaknesses in the existing regulatory and economic capital approaches, suggesting that banks must do more to better predict and provide for unexpected losses. Though a significant amount of discussion is currently occurring on how to best segment portfolios or predict key variables to fit the existing formulas, we believe that a re-examination of existing BV-MV assumptions with respect to credit risk or "*stressing the probability of default*" is required.

The paper develops a a tractable and empirically sound technique which is specifically tuned to the dynamics of retail credit card portfolios and which could be employed for either regulatory or economic capital. The key advantages of this approach are that it is based upon a much more accurate model of retail loan defaults, does not require any new data feeds, is based upon readily available modeling frameworks, and can adapt to portfolio changes such as those observed in the US financial crisis. The paper's PD-DTIR framework generates "*stressed* PDs" that can be used to derive loss rates used for provisioning of the Bank's capital requirement.

Using options theory we discuss the obligor's real option of default and subsequently develop estimates of expected losses from stressed PDs so as to determine loss provisions for consumer card portfolios held by financial intermediaries. Our method relies on the simulation of the obligor's probability of default distribution via changes in selected macroeconomic variables and the card holder's debt to income ratio. The methodology offers the flexibility of being tractable and scalable to data on the issuer's portfolio by geography and the credit card debt quality of the obligor.

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APPENDIX A

The Logit Transformation

The first step in defining the model for our data concerns the systematic structure, where the probabilities λ_i depend on the vector of observed covariates z_i in (6). The simplest idea would be to let λ_{it} be a linear function of the covariates, say

$$\lambda_{it} = z_{it}\beta \qquad \text{A.1}$$

where β is a vector of sensitivities discussed in (6). Expression A.1 is sometimes called the *linear probability model*. This model is often estimated from individual data using ordinary least squares (OLS). One problem with this model is that the probability λ_{it} on the left-hand-side has to be between zero and one, but the linear predictor $z_{it}\beta$ on the right-hand-side can take any real value, so there is no guarantee that the predicted values will be in the correct range unless complex restrictions are imposed on the coefficients. A simple solution to this problem is to *transform* the probability to remove the range restrictions, and model the transformation as a linear function of the covariates. We do this in two steps.

First, we move from the probability λ_{it} to the *odds*

$$\text{odds} = \frac{\lambda_{it}}{1-\lambda_{it}} \qquad \text{A.2}$$

defined as the ratio of the probability to its complement, or the ratio of favorable to unfavorable cases. Second, we take logarithms, calculating the *logit* or log-odds

$$\eta_{it} = \log(\lambda_{it}) = \log\frac{\lambda_{it}}{1-\lambda_{it}} \qquad \text{A.3}$$

which has the effect of removing the floor restriction. To see this point note that as the probability goes down to zero the odds approach zero and the logit approaches $-\infty$. At the other extreme, as the probability approaches one the odds approach $+\infty$ and so does the logit.

The logit transformation is one-to-one. The inverse transformation is sometimes called the *anti-logit*, and allows us to go back from logits to probabilities. Solving for λ_{it} in Equation A.3 gives

$$\lambda_{it} = \text{logit}^{-1}(\eta_{it}) = \log\frac{e^{\eta_{it}}}{1+e^{\eta_{it}}} \qquad \text{A.4}$$

APPENDIX B

The credit card portfolio accross an issuer's various credit classes or geograhic locations can be aggregated into one overall loss distribution. To accomplish this we begin by define expression (1) for each exposure class $l\left(l=1,2,...3\right)$ and each asset return as R_{it}^{l} ;

$$R_{it}^{l}=b^{l}F_{t}^{l}+\sqrt{1-b^{(l)2}U_{it}^{l}} \qquad \text{B.1}$$

where

$$F_{t}^{l}\sim N\left(0,1\right)\ \ U_{it}^{l}\sim N\left(0,1\right)$$

$\left(i=1,...,N_{t}.\ \ t=1,...,T\right)$ are normally distributed with mean $\left(\theta\right)$ zero and standard deviation $\left(\sigma\right)$ one. Idiosyncratic shocks U_{it} are assumed to be independent from the systematic factor F_{t} and independent for different borrowers. All random variables are serially independent.

The correlation between any two asset returns is then

$$corr\left(R_{it}^{l},R_{jt}^{s}\right)=\begin{cases}b^{(l)2} & l=s,i\neq j\\ b^{(l)}b^{(s)}\rho_{ls} & l\neq s,i\neq j\end{cases} \qquad \text{B.2}$$

where

$$\rho\equiv Corr\left(F_{t}^{l},F_{t}^{s}\right) \qquad \text{B.3}$$

Denotes the correlation between the random factors of two different credit classes. Usine these correlation estimates the loss distributions can be calculated by integrating over the joint distribution of the random effects. While th one dimentional integral from section 3 was numerically tractable, in general a higher dimentional integral requires a bit more sohistication, which can be acheived using Monte Carlo simulations.