Which Financial Measures Can Be Leveraged to Help Close the Unfunded Liability Gap for State Pension Plans?

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ABSTRACT

This paper attempts to identify the major determinants of the unfunded liability for state and local public defined benefit (DB) pension plans across the United States (U.S.) State and local pension plans are required to fulfill a commitment to pay lifetime pension benefits to its retirees. As the dependent variable, the unfunded liability (UL) within each state plan is analyzed to determine which financial lever emerges as the dominant force to help close the funding gap. Source data consists of 16 years (2003-2018) of Plan Data derived from the Public Plan Data (PPD) gathered by the Center for Retirement Research at Boston College (CRR) and the Center for State and Local Government Excellence (SLGE). Annual State Government Finances Tables published by the U.S. Census Bureau were also used to review state expenditures and revenues to include 1) Bureau of Economic Analysis/Real Gross Product Annual by State and 2) Trading Economics/U.S. GDP per capita. I apply a multiple linear regression empirical model of panel data with both cross-section and period factors fixed with robust standard errors. My findings show that the UL held a strong negative correlation to the discount rate, investment allocation, and the average 10-year investment returns. Model results also revealed that as the annual required employer contribution or the tax revenue per capita increases, the UL also increases. These results help to provide solutions on how certain financial measures can help reduce the unfunded liability gap (ULG) within state public DB pension plans across the U.S.

KEYWORDS: public pension plans; unfunded pension plans; unfunded accrued liability; defined benefit plans; defined contributions

JEL CLASSIFICATION H55; H75; H72; H55; G11

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Introduction

The main purpose of this study is to identify the key determinants of the unfunded liability (UL) for state and local public Defined Benefit (DB) pension plans. The UL is a measure of pension debt in plans across the U.S. This debt continues to rise while pension obligations to current and future retirees must also be satisfied. The UL is derived by subtracting the market value of plan assets from its accrued liabilities. If assets are less than liabilities, it signals a lack of funds set aside to cover all pension benefits and generates what is known as an unfunded liability on the financial books of many states across the country. If assets exceed liabilities, a plan is considered fully funded. The pension and employee benefits committee noted that full funding does not usually imply that the pension plan has sufficient assets to cover its solvency liabilities (unless the funding objective is to achieve a solvency level of funding.)[1] Throughout my manuscript, I will refer to the unfunded liability (UL) as the actual accrued liability and refer to the unfunded liability gap (ULG) as an expression of the financial factors that could help to reduce the liability. The contributing factors to receive a DB pension benefit, involves a combination of both the plan design and fiscal health of pension plans. Plan design can range from elements of retirement requirements such as age, years of service, average salary, service credits, mandatory employee contributions, while fiscal health includes parameters that measure the financial impact of the pension liability. Examples include and not limited to, employers' contributions, tax revenue per capita, Gross Domestic Product (Real GDP) per capita, budget surplus, discount rates and other investment measures.

Napoletoano and Schmidt (2021) highlights that retirement is a major concern that individuals have around their finances and points to the 2019 survey by the Aegon Center for Longevity that shows almost half of Americans worry that they will outlive their savings and are also concerned that their investments will not provide the returns needed to live comfortably in retirement. According to the survey, only 32 percent (global 25 percent) feel they are on track to achieve 67% of replacement income in retirement[2] To be well-situated and obtain financial freedom during retirement years, workers may choose to participate in either a defined contribution (DC) plan, or their employer may provide a DB plan. The choice of plan can depend on whether the employer is a private or public entity. Throughout this analysis, a public employer is defined as a state or an agency within the state, a city, county, township, school district, or a public institution of higher education. A private employer refers to an employer that is not

[1] Defined Benefit Pension Plan Funding and the Role of Actuaries Educational Monograph, May 2018.
[2] The Aegon Retirement Readiness Survey is conducted annually in collaboration with nonprofits Transamerica Center for Retirement Studies® in the United States and Instituto de Longevidade Mongeral Aegon in Brazil.

the state or any political subdivision, municipality, or other public agency of the state.[3] This report will draw attention to the defined benefit plans across the U.S. in the public sector.

To help better understand the impact of the pension liability within the United States. It is important to become acquainted with the recent retirement asset landscape as shown in Table 1 below. According to the Investment Institute Research Fourth Quarter 2019 year-end report, $32.3 trillion in total retirement assets of which, $6.7 trillion (21%) represent defined benefits plans that include federal, state and local plans and $3.4 trillion (11%) represent private sector defined benefit plans. The defined contribution landscape represents $9.0 trillion or 28% of total assets ($6.2 trillion represent private DC plans and $2.7 trillion represent 403(b) and 457 plans.) State and local government pension benefits are paid from trust funds to which public employees and their employers contributed while they were working (PPD database, 2019). The assets within the trust fund are expected to grow over time and are used for funding a lifelong pension obligation for current and future retirees. The assets are measured against the pension liabilities to determine whether a plan is unfunded (lower assets than liabilities) or fully funded (higher assets than liabilities). The relationship between financial factors (such as projected contributions, economic measures, portfolio allocation and investment returns) and the UL are observed throughout this paper.

The defined benefit (DB) obligation in the public sector represents a mathematical calculation of an estimated percentage of base salary and a monthly annual plan benefit. This benefit commonly includes three key components for normal retirement such as age, salary, and length of service (Ashford and Schmidt, 2021). For example, the State of Connecticut's Teachers Retirement Board estimator of benefits includes several factors such as age at retirement, number of years worked, (i.e., credited service) and three highest salary years (Appendix, Table 2). Many private companies have shifted from DB to DC plans to transfer the risks from the employer to the employee (Broadhurst, Palumbo and Woodman, 2006). The rationale for this change by many private firms has been due to a few key and notable facts such as the elimination of the future pension liability owed to plan participants, tax benefits, lower plan administration expenses and employers no longer responsible for the investment performance of the plan. The employee bears the financial risk of participating in a DC plan where the amount of savings at retirement is determined by factors such as age, time horizon, and most importantly, the contribution amount and investments selected. For DB plans, the employer bears the investment risk that returns on assets may fall short of the growth of the pension liability. Longevity risk is another risk that employers may realize because they are obligated to offer DB benefits as a deferred lifetime annuity. As

[3] Wage and Hour Division, an agency within the U.S. Department of Labor, Washington, DC 20210, www.dol.gov, 2020.

beneficiaries of DB plans now live longer, it will also extend the benefit payment timeline. Therefore, it is impossible to ignore DB plans in the public sector where states (the employer) and ultimately taxpayers across the U.S. continue to bear the risk on behalf of the state and local employees.

TABLE 1

U.S. RETIREMENT MARKET ASSETS[4]
$32.3T
December 31, 2019

Type of Plan	Type of Entity	Plan Contributions	Who Bears Most? Risk?	How is the monthly benefit determined?
DB Defined Benefit $10T[5]	Public and Private ($6.7T/$3.4T)	Employer and/or Employee	Employer	Benefit is calculated in advance: based on the employee's years of service, age, salary etc.
DC Defined Contribution $9T[6]	Public and Private ($2.7T/$6.2T)	Employer and/or Employee[3]	Employee	Benefit is undefined and controlled by the employee factors such age, time horizon, amount contribution and funds selected
Individual Retirement Accounts (IRAs) $11.0T[7]	Financial Institutions	Self-Directed by Investor	Investor	Benefit is undefined and controlled by the investor factors such age, time horizon, amount contribution and funds selected

Retirement income can be generated from a variety of sources such as a pension or annuity, personal savings, investments (i.e., CDs, bonds, mutual funds, stocks and real estate holdings, etc.) and Social Security benefits.[8] Social Security is a major source of retirement income for those wage earners that are covered employees and according to the Social Security Administration (SSA), 89 percent of U.S. workers ages 21 to 64 are in "covered" employment. There are some groups that are considered "non-covered" employees and in many cases, these employees will be granted a state pension plan rather than Social Security. Non-covered employees may include teachers, U.S. government employees hired before 1984, railroad employees covered by a separate pension and foreign nationals who work in the U.S. for their home government.[9] The livelihood and employee benefit expectations of the worker were seriously impacted within both the private and public sector. In fact, the 1980s brought a drastic shift to the traditional employer/employee relationship with employers using layoffs to maximize profits. Although

[4] Investment Institute Research Fourth Quarter 2019
[5] Defined Benefit plan provide a fixed, pre-established benefit for employees at retirement, IRS.gov, 2021.
[6] Defined Contribution plan is a retirement plan in which the employee and/or employer contribute to the employees individual account under the plan, IRS.gov, 2021
[7] A Traditional IRA is a tax-advantaged personal savings plan where contributions may be tax deductible, IRS gov, 2021.
[8] The Financial Industry Regulatory Authority (FINRA) is a nongovernmental organization that writes and enforces rules for brokers and broker-dealers, Finra.org, Sources of Retirement Income, 2021.
[9] Formerly called the American Association of Retired Persons, AARP Social Security Resource Center, is a nonprofit, nonpartisan organization that empowers people to choose how they live as they age, https://www.aarp.org/retirement/social-security/questions-answers/benefits/.

public sector workers were impacted to a lesser degree, there is no doubt that state and local entities (including teachers and schools) have been impacted by the changing socio-economic times with increasing occurrences of job eliminations and school closings. According to the Bureau of Labor Statistics (BLS), the median tenure of workers ages 45 to 54 is 7.5 years in 2020 as compared to 8.4 years in January 1981.[10]

Employer and/or employee contributions made into some type of employer-sponsored retirement plan are needed to help satisfy income for a retiree's lifetime. The employer makes most contributions to a DB plan however, employee contributions can be required, or voluntary contributions may be permitted. For DC plans, the employees can choose to voluntarily contribute to their retirement plan up to a certain limit (401k plans). Their employers can choose to match employee contributions with additional funds or choose to make elective contributions irrespective of the amount contributed by the employee (Enright, 2021).

According to the aggregated state and local pension DB plans across the U.S. from 2003-2018, adjusted PPD data increased the unfunded liability to $1.43 trillion in 2018 from $1.40 trillion in 2017 (Fig. 1). The UL is designated as the key dependent variable; and multiple independent variables are measured to determine the relationship between the unfunded liability across all 50 states. Independent variables represent annual plan contributions, tax revenue, budget surplus, GDP per capita, discount rates and others were tested against the UL. There is much discussion around whether carrying an unfunded pension liability impedes a public employer's (i.e., state or employer) ability to continue to pay millions of current and future retirees their respective pension benefits. One position claim that having an outstanding UL does not result in a state's incapability to fund its pension plan (Florida Public Pension Trustees Association, FPPTA, 2011). The opposing position states that being underfunded[11] is a hindrance and could have a negative long-term impact on the state's pension obligation (Novy-Marx and Rauh, 2009.)

[10] State and Local Governments Under the Fair Labor Standards Act (FLSA). U.S Department of Wage and Labor division, March 2011.

[11] According to the Florida Public Pension Trustees Association (FPPTA), "Understanding Public Pension Plan's Unfunded Liability." The term "underfunded" means that the plan sponsor has not made sufficient contributions to fund the present and future liabilities of promised benefits.

Brief History

Pensions initially served as extra compensation to persuade people to enlist into the military (Longley, 2020). The first pension law was enacted during the Revolutionary War, and initially, states were responsible for making payments and many were faced with budget challenges. In 1789, legislation was passed where the U.S. government assumed full responsibility for making the pension payments to disabled veterans. The federal government passed additional legislation increasing benefits for veterans and their families as the country grew ever more prosperous. In 1818, benefits were expanded to all veterans for life instead of a few years. In 1875, The American Express Company was the first private company to offer a DB pension plan to help fulfill the need for lifetime retirement income. The plan was offered during the Reconstruction era to workers who were 60 years of age, had a tenure of 20 years and were directed to retire by a manager (Phipps, 2021). Twenty years later, public non-military plans were offered, and the Chicago Teacher's plan became one of the oldest non-military public plans with an inception date of 1895. The state of Illinois along with several plans within CA, MA, MN, NY, and PA were established during the Progressive era which took place during the 1890s to 1920s (Longley, 2020).

The Employee Retirement Income Security Act of 1974 (ERISA)[12] was put in place to establish a regulatory framework for private DC and DB plans. These private pension plans provided financial support in the retirement years for many Americans and ERISA was Congress's attempt to devise a comprehensive regulatory program to protect millions of Americans (Wooten, 2004). ERISA plans must also adhere to Department of Labor regulations under the Employee Retirement Income Security Act. ERISA rules help mitigate certain risks for private DB plans such as agency risk, forfeiture risk and default risk (Wooten 2004). Agency risk is the risk that the employer might misuse assets and not have enough to meet their pension obligation. Forfeiture risk is the risk that employees could lose their pension benefit due to layoff or job change. Default risk is the risk that the plan could become insolvent and be unable to fulfill its financial commitment. These key risks currently exist in the public pension world today. However, mitigation of these risks in the private sector includes but are not limited to:

- Fiduciary standards of conduct, minimum vesting, and funding standards for employers.
- Employers are required to set up separate financial resources to help meet their obligation.

[12] The Employee Retirement Income Security Act of 1974 (ERISA) is a federal law that sets minimum standards for most voluntarily established retirement and health plans in private industry to provide protection for individuals in these plans, Employee Benefits Security Administration, an agency within the U.S. Department of Labor.

- Participation in a federal program (Pension Benefit Guaranty Corporation – PBGC) that pays vested pension benefits in the event of plan termination in the private sector DB plans.[13]

Public DB plans (non-ERISA plans) are not required to follow ERISA standards. For example, no insurance (PBGC) exists for public employees. Therefore, should a gap in funding the liability exists, the state and/or municipality should reassess their budgets to find ways to cover the shortfall. My findings reveal that an increase in annual required contributions (ARC) would not necessarily decrease the unfunded liability but would increase the dependent variable. Although this was an unforeseen finding, there is no mandate for public state plans to make timely contributions. In fact, the ARC is perceived to be an optional or suggested payment by public DB plan administrators and not truly as a required annual payment. There is no mandate for public state plans to make timely contributions at all. This is remarkably similar to how households manage their day-to-day budget. Individuals choose to determine the amount, timing, and frequency of their respective bill payments. Inconsistent behavior may result in consequences such as fee penalties, higher interest rates and a reduced credit score. Similarly, state, and public employees can skip making their share of contributions. The expectations of employees do vary from state to state and there are situations where employee contributions are either voluntary or mandatory. As the normal cost of benefits for each member continues to be accrued, a continual pattern of nonpayment and/or erratic payments would cause an increase in the unfunded liability.

As a former investment consultant who acted on behalf of insurance company retirement providers for many years, I have witnessed public pension administrators throughout the U.S. who were called upon to make key decisions for both Defined Contribution (DC) and Defined Benefit (DB) retirement plans. These key overseers usually consist of several board members that hold roles such as Human Resource Directors, Finance Directors and Chief Financial Officers (CFO). It is also common for board members to gain membership through appointment. the board representatives have an important responsibility to partner with a provider of retirement services (i.e., insurance companies, investment consultants, brokerage firms and mutual fund companies) as well as an actuarial firm to make key investment decisions for thousands of employees. Most boards follow the guidance of an investment policy statement (IPS) which outlines parameters around investment selection and the addition and removal of investments for DC and DB plans. This process is in place to ensure that retirement plans are properly positioned to meet their future pension obligations as employees retire from their employer. DB plans take a similar approach as their DC counterparts in that both strategies seek to find the best asset allocation to achieve

[13] Pension Benefit Guaranty Corporation, A U.S Government Agency, updated March 2021, https://www.pbgc.gov/about/faq/pg/general-faqs-about-pbgc.

the most efficient return. Public and private entities must follow the standards and the plan reporting requirements of the Financial Accounting Standards Board (FASB). The objective of FASB and the Governmental Accounting Standards Board (GASB) is for plans to report benefit obligations and pension fund assets in a uniform manner.[14]

Literature Review and Motivation

As the industry examines the plan design and financial health of public pension plans, notable statistics studied by actuaries, pension administration, consultants and other industry experts were prevalent in the pension liability conversation. Antolin (2008) in collaboration with other private institutions assessed performance by country on a risk-adjusted basis using relatively standard investment performance measures. Additionally, the relationships between the characteristics of each pension system were examined including individual regulatory environments and the investment performance. Antolin highlights a few challenges and limitations with reporting and analyzing numbers by country such as the validity of the risk-adjusted measure Sharpe ratio and the use of artificially constructed country-specific benchmarks for a large variety of pension plans across the countries (not necessarily the same). His analysis included both DC and DB private plans and lacked attention on public pension plans. Another noteworthy study by Beshears, Liabson and Madrian (2011) finds that replacement income within the public sector varies greatly across jurisdictions. Though a reduction of the unfunded pension liability was not of emphasis, they specified that the shift from DB to DC plan offering would result in the need for participants to save more in other retirement accounts. The authors also make a distinction on how public sector and primary DC plans are characterized by required employee or employer contributions vs. employee voluntary contributions within the private sector. Their research concludes by applying lessons from savings behavior in private sector savings plans to the design of public sector plans.

Budget Stabilization Funds (BSFs) for public defined benefit plans are reserves set aside to protect the state budget from cyclical changes in revenue and expenses. Clairs (2013) analyzed deposit and withdrawal activity and its link to the employer contribution. His empirical results suggest that BSFs with strict deposit rules are associated with higher pension contributions, while strict withdrawal rules are associated with lower contributions. The connection discovered between the BSF, and pension contributions aided in my decision to select employer contributions as a primary independent variable and compared to the unfunded liability.

[14] The Governmental Accounting Standards Board (GASB) is the independent, private- sector organization based in Norwalk, Connecticut, which establishes accounting and financial reporting standards for U.S. state and local governments that follow Generally Accepted Accounting Principles (GAAP), September 2021.

Novy-Marx and Rauh (2009) explores the true funding status of public pension plans across the U.S. and, at times, question the usefulness and loosely reported pension liability. According to Novy-Marx and Rauh (2009), each state plan reports only one actuarial number for its pension liability. They suggest that, at a minimum, states should be required to report liabilities under several pre-specified discount rates, such as Treasury interest rates and interest rates on taxable municipal bonds. The impact of discount rates on the unfunded liability was not explored in this manuscript. Other non-academic writings including Mohan and Zhang's (2011) from University of Dayton School of Business, the authors studied determinants of public pension plan risk-taking behavior using the percentage of total plan assets invested in the equity markets and the pension asset beta as measures of investment risk. They found that government accounting standards strongly affect public fund investment risk, as higher return assumptions (used to discount pension liabilities) are associated with higher equity allocation and beta. The link between whether a link exists between discount rates and the unfunded liability will be explored in this paper.

The condensed report (PEW and the Arnold Foundation, 2014) highlights that, up through 2012, pension plans progressed from fixed income to an increased investment in equities and alternative funds. The report also suggests that such changes come with increased costs and uncertainty around the future realized returns. The assumed rate of return was compared to the 30-year Treasury bond. It was noted that, between 1992 and 2012, the difference between the assumed rate of return and the yield on 30-Year Treasury Bonds has increased from .33 percentage points to 4.83 percentage points. This brief and high-level analysis alluded to the fact that there has been a shift to riskier investment allocations through 2012. To further this study research, my review will include the investment allocation by state level to determine whether a shift to equity and alternative investments continue to exist.

The Annual Required Contribution

The annual required contribution (ARC) is a principal factor when determining the true cost of the pension plan.[15] The annual required contribution reflects the employer's contribution as reported in the required supplementary tables for GASB accounting purposes. There are three main elements that comprise the total expected contribution such as the employer normal cost amount, amortization of the UL, other expenses (retirement, medical and administrative) and interest expense on the prior year's pension obligation. The annual required contribution is guided by General Accepted Standards Board (GASB) statements 67 and 68 in the U.S. and there are factors (investment return assumptions,

[15] " Understanding Public Pension Plan's Unfunded Liability," Florida Public Pension Trustees Association (FPPTA), p. 4.

amortization methods, funding policies and timing of payments) that differ from plan to plan and can impact its calculation[16].

One may presume the ARC is a requirement for DB public plans because it is described as 'required,' however, it is generally known that there is an expectation for payment to be made versus a requirement that it must be made. As an important distinction, this optional approach has resulted in partial contributions to fund present and future liabilities or skipped contributions for several years. Although making a payment should normally reduce the rate of indebtedness, it can actually increase the rate since; 1) the cost of the plan administration continues to be charged to the debt and 2) interest on the unfunded liability continues to be applied. If a public plan chooses not to make the full ARC payment on an annual basis and either makes partial payments, no payments or pays erratically, the clock will continue to tick and increase the liability/debt of the total plan.

The ARC calculation is an annual snapshot in time of the DB plan. If the plan sponsor does not pay its ARC in full every year, no amount of payment (unless it is in full) will stop the unfunded liability from continuing to increase. Like the private sector DB plans of old, an ERISA-like requirement must be passed where all plans must pay their ARC into their plan every year. This is the one of the ways plans public DB plans will have hope of funding the promise made to their employees.

As discussed earlier, private DB plans adhere to regulatory rules of the Employee Retirement Income Security Act (ERISA). Additionally, the extension of ERISA, known as the Pension Protection Act (PPA) of 2006 requires private plans to become 100% funded over time and to adhere to financial assumptions that pension plans must fund objectives. These measures were established to ensure that minimum contributions are made to satisfy the annual contribution expectation. A more recent change to single employer private DB plans is the calculation for discounting interest rates. In response to a period of low interest rates, Moving Ahead for Progress in the 21st Century Act (MAP-21), established a process for determining minimum and maximum interest rates for discounting based on 25-year averages of historical corporate bond yields (Topoleski and Myers, 2020.) This act suggests that a lower interest rate would result in a lower assumed investment return and/or more realistic present value of future payments for employers to meet their pension obligations. As originally established, the funding corridor was

[16] Established in 1984, the Governmental Accounting Standards Board (GASB) is the independent, private- sector organization based in Norwalk, Connecticut, which establishes accounting and financial reporting standards for U.S. state and local governments that follow Generally Accepted Accounting Principles (GAAP), https://www.gasb.org/aboutgasb.

scheduled to widen eventually, which, when applied to the specified interest rates, would have resulted in the use of lower interest rates to calculate private DB pension obligations.

Data and Methodology

The Public Plans Data (PPD), United States Census Bureau, Bureau of Economic Analysis and Federal Reserve Economic Data (FRED) were the four main data sources needed to analyze the relationship between the dependent and independent variable by state.

The PPD was the leading data used and comprised of annual information on the largest state/local pension plans in the U.S. The data is produced by the Center for Retirement Research at Boston College (CRR) and the Center for State and Local Government Excellence (SLGE) represented over 3,700 observations from 2001-2018 across all 50 states and the District of Columbia. Plan data does not track federal employee retirement plans and therefore those plan types were not included in this analysis. Key pension plan data included assets, funding, investments, other vital metrics, and membership within each state across the U.S. In fact, data was available for multiple plans within each state from 2001-2018. To obtain a balanced data set, where each plan is observed during the same time-period, 2001 and 2002 data were excluded. A full comprehensive sample of data representing 2003-2018 was analyzed. All 50 states were represented, and the federal District of Columbia was excluded because 16 years of plan data were not reported. Data for 2019, was not yet complete and hence 2003 through 2018 represented 2,800 observations. On average there were approximately 3.5 public retirement plans per state (including plans for teachers, municipalities, state workers etc.) One plan was assessed for Hawaii, Idaho Mississippi, Oregon, and Wyoming. California and Texas represented the highest number with 15 and 10 public plans, respectively. Data from the plans' financial documents at times were used to highlight plan facts. The PPD consists of public pension plans aggregated across all states and reflects significant variations in plan benefit design, plan funding, membership composition, and investment strategies. It is an appropriate starting position to analyze the plan results at the macro state level. Disaggregation could be a suitable next step for future research (PPD database, 2019). Each state's Gross Domestic Product (GDP) per capita was used to determine its relationship with the unfunded liability. The Bureau of Economic Analysis was the main source for the annual Real GDP and the United States Census Bureau was used to determine the per capita numbers by state for GDP and tax revenue measures. Additionally, both sources were also used to assess the states' budget/ deficit/surplus in two ways, (GDP/per capita and GDP/per state). The budget deficit/surplus by each state was reviewed to determine its relationship against the UL. The U.S. Census Bureau Annual Survey of State Government Finances Tables were used to provide both the revenue and expense data to calculate the budget deficit or surplus per GDP/state and GDP/per capita.

Empirical Model

The multiple ordinary least squares linear regression model was selected because it is one of the best models to assess the strength of the relationship between the unfunded liability (dependent variable) represent by RUFL and fifteen predictor variables (independent variables) across the US. Independent variables include, expected contributions, economic methods (i.e., tax revenue and budgets) and investment strategy (assumed investment returns, actual returns, and portfolio mix).

Multiple ordinary least squares model:

$$\mathbf{RUFL}_i = \beta_0+\beta_1 ARCR\_ER_{it}+\beta_2 ARCR_{it}+\beta_3 TAXRPC_{it}+\beta_4 GPERC_{it}+\beta_5 BUDGDP_{it}+\beta_6 BUDPC_{it} +\beta_7 DISCR_{it}+\beta_8 PTRTN_{it}+\beta_9 EQRTN_{it}+\beta_{10} FIRTN_{it}+\beta_{11} A3YR_{it}+\beta_{12} A5YR_{it}+\beta_{13} A10YR_{it}+\beta_{14} EQALL_{it}+\beta_{15} FIALL+\varepsilon i$$

Where,

RUFL= is the dependent variable -referred to as return on unfunded liability [unfunded liability as a % of assets]
$ARCR\_ER_{iti}$ =Employers projected actuarial required contribution (ARC rate as a % of payroll)
$ARCR_{it}$ =[Employer + Employee] projected actuarial required contribution (ARC rate as a % of payroll)
$TAXRPC_{it}$ =Tax revenue per capita
$GPERC_{it}$ =Gross Domestic Product (Real GDP) divided by Per Capita= [Real GDP by State/Per Capita]
$BUDGDP_{it}$ =Total Revenues (TR) less Total Expenditures (TE) divided by GDP per State= [TR-TL/GDP]
$BUDPC_{it}$ =Total Expenditures (TE) divided by GDP per Capita = [TR-TL/GDP Per Capita]
$DISCR_{it}$ =Assumed Discount Rate used to value the current cost of future pension obligations and is determined by estimating expected rates of return.
PTRTN= Portfolio Return is the overall return of each plan includes various asset allocations
EQRTN= Equity Investment Return
FIRTN= Fixed Income Investment Return
A3YR= Average 3-Yr Investment Return
A5YR= Average 5-Yr Investment Return
A10YR= Average 10-Yr Investment Return
EQALL= Equity Asset Allocation Mix
FIALL= Fixed Income Allocation Mix

Foundational structure

Multiple Linear Regression Model

Table 3 below shows the structure of the multiple linear regression model. The dependent variable is equal to the accrued liability minus actuarial assets and expressed as a % of assets (RUFL). The independent variables, mapped in three main categories such as expected contributions, economic methods, and investment strategies. The investment strategy category has been further divided into three separate groups to highlight a few differences between assumed returns, actual returns, and portfolio mix. As a result, five groups of independent variables were formed and analyzed.

Each group were identified as balanced or unbalanced. Balanced data shows that every cross-section follows same regular frequency, with the same start and end dates. For example, the balanced data consists of 150 cross sections in the main model with annual data from 2003-2018. The unbalanced data represent irregular or unreported data. For instance, the Atlanta General Employee Pension Fund showed missing data for portfolio returns for 2018. Overall, there were 1,442 unbalanced observations in group 4 while group 5 revealed 2,302 observations.

Table 3 shows the expected payment known as Annual Required Contribution (ARC) is the main independent variable and placed in Group 1. The "ARCR_ER" solely represents the employer contribution and expressed as projected actuarial required contribution (ARC_ER rate as a percentage of payroll) as shown on Fig. 4. The second variable in Group 1, identified as ARCR represent a combination of both employee and employer contribution. This method was employed to determine whether the ARCR_ER would act differently if an employee contribution were included in the payment. Group 2 represents other key economic measures such as tax revenue per capita, GDP per capita, budget per capita and budget as a percentage of GDP. Group 3 include an assumed investment return (discount rate) as a solo variable. Group 4 illustrates average 3-year, 5-year, and 10- year rates of return as well as portfolio investment returns. Finally, group 5 represents the portfolio mix allocations (i.e., equity vs. fixed income).

Table 3

Dependent variable	**RUFL= Unfunded Liabilities= Expressed as Assets less Liabilities divided by Assets**				
Independent variables:	**Financial Levers**				
	Expected Contribution(s)	**Economic Methods**	**Investment Strategy**		
	Group 1 Balanced	**Group 2 Balanced**	**Group 3 Balanced**	**Group 4 (Unbalanced)**	**Group 5 (Unbalanced)**
			Assumed Investment Returns	**Actual Investment Returns**	**Portfolio Allocation Mix**
	ARCR_ER[1] **ARCR[2]**	**TAXRPC[3]** **GPERC[4]** **BUDGDP[5]** **BUDPC[6]**	**DISCR[7]**	**PTRTN[8]** **EQRTN[9]** **FIRTN[10]** **A3YR[11]** **A5YR[12]** **A10YR[13]**	**EQALL[14]** **FIALL[15]**
Other factors	I= cross plan id t= year				

All Preliminary Variables Tested and Explained

Table 3 list the dependent and all independent variables initially assessed throughout the analysis; represents the aggregate of all plans by state from 2003-2018

[1]Employers projected actuarial required contribution (ARC rate as a % of payroll)
[2]Total [Employer + Employee] projected actuarial required contribution (ARC rate as a % of payroll)
[3]Tax revenue per capita
[4]Gross Domestic Product (Real GDP) divided by Per Capita= [Real GDP by State/Per Capita]
[5]Total Revenues (TR) less Total Expenditures (TE) divided by GDP per State= [TR-TL/GDP]
[6]Total Expenditures (TE) divided by GDP per Capita = [TR-TL/GDP Per Capita]
[7]Assumed Discount Rate used to value the current cost of future pension obligations and is determined by estimating expected rates of return.
[8]PTRTN= Portfolio Return is the overall return of each plan include various asset allocations.
[9]EQRTN= Equity Investment Return
[10]FIRTN= Fixed Income Investment Return
[11]A3YR= Average 3-Yr Investment Return
[12]A5YR= Average 5-Yr Investment Return
[13]A10YR= Average 10-Yr Investment Return
[14]EQALL= Equity Allocation Mix
[15]FIALL= Fixed Income Allocation Mix

Model Results

Results are described by each type of financial measure such as contributions, economic levers, and investment strategy. Counterintuitive results for both determinant variables (ARC and tax revenue per capita variables) were observed. There was an expectation for the UL to decrease as these variables increased. However, results show that a 1% increase in the ARC by the employer or tax revenue will increase the UL. The multiple linear regression shows three independent variables that emerged as highly statistically significant: a) annual employer contribution rate (ARC_ER), b) tax revenue per capita and c) equity allocation mix. The results of the 15-variable linear regression are shown on Table 3A in five groups. Those variables that were not statistically significant were removed and the model was condensed from five groups to two groups (Group 1 balanced and Group 2 unbalanced). The results are shown on Table 3B. Fig. 1, displays the total aggregated unfunded liability which is the dependent variable as the total value vs. the unfunded liability as a percentage of assets from 2003-2018. The panel model was adjusted to reflect only those plans that showed data in each year from 2003-2018.

Model Results-Contributions

A 1% increase in the ARC_ER (expressed as a percentage of payroll) will increase the unfunded liability (expressed as a percentage of assets) at 2.0% (Table 3A and 3B). For example, if the 2018 employer required annual contribution for California Public Employees Retirement Fund (California PERF) increased by 1% from $19.90B to $20.10B, the unfunded liability will rise from $150.40B to $153.41B.[17]

Findings show that a slight increase to ARC_ER may not cause a reduction but an increase to the UL. One may suggest that the rate of indebtedness should be reduced by solely making a payment however, the results show that the rate can actually increase. This increase could happen as a result of plan administration costs and the interest charged on the unfunded liability as time moves forward. A key learning is that the ARC calculation determined by actuaries is an "expected" contribution, not a required contribution for public pension plans. Some states have a policy that requires payment of the ARC,

[17] Public Plan Database (PPD) (2019) Public plans data. Available at https://crr.bc.edu/data/public-plans-database, Google Scholar. Adjusted calculations were used to determine the impact to the unfunded liability.

however other factors can affect those policies and the actual payment of the ARC. For example, in New Jersey, two separate legal rulings found that the state's ARC requirement, set in statute, did not constitute a "self-executing appropriation" (Brainard and Brown, 2015). Therefore, should a public plan choose not to make the full ARC payment on an annual basis or make partial payments, no payments or pays erratically, the debt or liability will continue to increase. This behavior can be compared to the process of making of credit card payments, for example, if a minimum payment is made on a $500 outstanding balance, the minimum payment gets absorbed by the huge interest rate that is continued to be charged, in addition the annual credit card fee must be paid.

The public pension plan itself does not stay static, there are long-time employees who were eligible for a defined benefit pension now retiring and receiving payouts that must be funded on an annual basis. At the same time, new employees are being hired (and not all are necessarily in their 20s or 30s), and therefore new retirees add to the unfunded liability calculation, thus also inflating the debt or liability of the plan.

The current approach gives a plan complete freedom to choose whether to contribute the full expected contribution amount, to make a partial payment of that expected amount or to decline to contribute at all. I decided to further assess the model with a one-year lag to determine whether year 1 ARC_ER would help impact funding in year 2. Table 4A results showed an increase to the liability where the rate shifted from 2.0% without a lag to 1.9% with a one-year lag. Results were and is highly statistically significant. A further reduction is shown in year 2 at 1.8%. As an example, if we apply the ARCR_ER rate of reduction to the California PERF plan, the UL will rise to $153.39B with a one-year lag and $153.09B with a two-year lag. Overall, both the first- and second-year lag shows statistically significant results and reveals that the impact of a reduction may take a few years. It is important to note that results show that the UL begins to slightly reduce at .52% when employee contribution is included with the employer contribution [ARCR]. Table 3B and 3C shows that this reduction occurs without any lags.

Model Results-Economic Levers

As the tax revenue per capita increases by 1%, the UL will increase by more than 1% (Table 3B). State and local government pension benefits are paid from trust funds to which public employees and their employers contributed while they were working, not from general operating revenues. Trust fund assets are invested and grow over time[18]. Tax revenues are another sourcing option for state public pension plans, to help fund their obligations. However, unlike private plans, there are no minimum funding

[18] Public Plan Database (PPD) (2019). Available at https://crr.bc.edu/data/public-plans-database, Google Scholar.

requirements for the UL and therefore the independent variable is likely to continue to increase should the plan decide against using tax revenues to make payment towards the annual contribution. A possible solution is to mandate a certain percentage of state tax revenue should be used to fund the pension trust obligation. Bagchi examined the effects of political competition on the generosity of public sector pension plans and finds that as the level of political competition in a municipality increases, pension plans become more generous, and plans may not fully fund their plans to keep taxes low (Bagchi, 2019). In a more recent example, due to the 2020-2021 COVID-19 pandemic, many states' tax revenues were reduced, and those that use tax revenue to help pay for the unfunded liability chose to either skip payment or make a partial payment toward their pension contribution. Others were waiting to see what amount of assistance would arrive from the federal government. California canceled $500 million from a $3 billion payment authorized in 2019 to pay down unfunded pension liabilities through fiscal 2023 and redirected $2.4 billion remaining from the initial allocation to instead pay pension contributions owed by school districts and community colleges [19]. Tax revenues showed counterintuitive results and are another sourcing option for state public pension plans, to help fund their obligations. As tax revenues increase by 1%, the UL is likely to continue to increase if the plan decides against using tax revenues to make payment towards the annual contribution. Table 4A shows that, at a 1% rate increase in tax revenue per capita with a lag of two-year will create a substantial increase the UL from 1% to 7%. The results clearly show that until public plans are required to use tax revenue dollars to pay down on the UL the liability, the debt will continue to increase even further in the second year. The second-year lag results show less than a 10% risk that there is enough evidence to reject the null hypothesis which states that no relationship exists between state pension plans and the tax revenue per capita.

Model Results-Investment Strategy

The investment strategy involves several levels of assessment. To reduce pension debt, a higher market value of plan assets must be achieved to cover the cost of current and future liabilities. Results show that when the portion allocated to investment in companies (expressed as EQALL) increases by 1%, the UL is reduced by .18% (Table 3B). While some data is unbalanced, the equity mix represent 1,520 records and generates a mean of 52%, median 55% and standard deviation of 10.8%. The one-year and two-year lags for equity allocation continue to show a reduction in the UL at around .17% and .28% respectively, with strong results particularly in the second year (Table 4A). These statistics reveal that public defined benefit

[19] How Pandemic-Driven Revenue Shortfalls Could Affect State Pension Contributions, Pew Research Center, Washington, D.C. January 13, 2021, https://www.pewtrusts.org/en/research-and-analysis/articles/2021/01/13/how-pandemic-driven-revenue-shortfalls-could-affect-state-pension-contributions/Google Scholar.

plans have shifted from the assumption that fixed income represents the larger percentage of the investment mix (PEW and the Arnold Foundation, 2014). Excluding 2008, assets were also allocated to real estate, alternatives, private equities, commodities, and cash. The 2008 allocation shows that the financial crisis has certainly impacted the allocation mix selection of many of the public pension plans across the United States. Fig. 5, illustrates the trend of both equity and fixed allocation aggregated by all plans used in dataset over the 16-year testing period. Though the fixed income allocation mix was not significant, the investment results for conservative investments (bonds, certificate of deposits etc.), and the 10-year average returns were all statistically significant. A 1% increase to the fixed income return would increase the UL by .27% (Table 3B). This could occur if returns generated from stable assets do not cover the outstanding liabilities between 2003-2018. While Table 3A shows a 1% increase to the equity return portfolio will reduce the UL by a minute percentage, the results were not significant. The reduction to the UL does occur however, with an increase in the 10-year investment returns. The longer-term return (average 10-year) is an essential factor to help close the ULG. As the 10-year return increases by 1%, the unfunded liabilities would reduce by 2.6%, results were highly statistically significant (Table 3B). The one-year lag result for the 10- year return is also highly statistically significant and shows a 3.0% reduction in the UL (Table 4A). These results tell us that the investment strategy of developing the right mix of equities and fixed income investments could overtime provide the returns needed to help reduce the UL. Fig. 2 shows a snapshot of the average 10-year return of all aggregated plans used in the dataset from 2003-2018. Note that 2004 shows the highest average 10-year return of 9.7% vs. 2009 and 2010 lower returns of 3.1% and 3.5%.

Fig. 6, shows both equity and fixed returns aggregated by all plans used in the dataset over the 16-year testing period. It is important to observe that twelve out of sixteen years (75% of the time), higher equity returns were generated versus fixed returns. Hence, the investment selection committee may choose to increase their risk tolerance and allocate more to equities with hopes to achieve a higher return. This action along with the ability to make timely ARC payments could help reduce the ULG.

Model Results-Assumed Discount Rate

The assumed investment return (commonly referred to as the discount rate) is important when determining an effective investment strategy for public pension plans (Fig. 3). The discount rate is used to value the current cost of future pension obligations and is determined by estimating expected rates of return. This rate is a function of the risk-free interest rate plus the risk premium associated with public plans not being able to pay their pension obligation and should reflect the risk of the pension liabilities (Bui and Randazzo, 2015). The public plan aggregated data across the U.S., shows an average assumed

discount rate of approximately 7.79% from 2003-2018. However, rates used by plans, may vary across the country. For example, according to the PPD, the Connecticut State Teachers' Retirement System reported discount rates of 8.50%, 8.25% and 8.00% respectively from 2016-2018.

This shows a slight reduction in its discount rate over three consecutive years. The Connecticut State Employees Retirement System used a lower rate of 6.90% from 2016 through 2018. Wisconsin, a 100% funded ratio (considered a fully funded plan[20]) shows a discount rate of 7.2%, 7.2% and 7.0% from the same period (2016-2018). Pennsylvania Municipal Retirement System, also 100% funded according to the Pension data, uses a discount rate of 5.5%, 5.25% and 5.25% from 2016-2018.

Josh Rauh, chief economist at Stanford Graduate School of Business, states that public pension plans are using an assumed discount rate that is too high. Rauh (2016) suggests a lower rate be used which would mean that more money would need to be contributed now to support pension benefits that will be paid out in the future. Some critics argue that by using an interest rate that is unreasonably low makes pension funds worse than they are (Bond, Tyler, 2016). The use of a higher assumed investment return would result in a higher present value of future payments of pension benefits. Fig. 3 compares the assumed investment return known as the discount rate to the actual average 10- year return. The average 10-year return and the discount rate represent aggregated plans used in the dataset from 2003-2018. The discount rate shows a consistent average of 7.8% vs. the average10-year return of 6.8%.

A uniform and consistent proxy such as 10-year Treasury notes, 30-year bonds or high-grade municipal bonds should be set as a standard for calculation of the assumed investment return. The rates would be lower and may have a more realistic asset value than the assumed discount rate disclosed in the data provided. Persistent testing of the discount rate shows a larger reduction of the UL without a lag than with a two-year lag. Results reveal that a 1% increase in the discount rate will reduce the UL by 14% with a no lag (Table 3C). This is a significant drop to the UL and the reduction will help to close the UL gap. It may also explain that higher assumed investment returns reported by plan actuaries could result in a lower UL. When evaluated with other statistically significant variables with one and two-year lags, findings show that the one-year lag result was not statistically significant. However, the two- year lag reveals that a 1% increase in the discount rate will cause the UL to fall by 3.68% with a two-year lag (Table 4A). The results are statistically highly significant. To put it in further context, if the discount rate increases by 1%,

[20] According to Florida Public Pension Trustees Association (FPPTA) a plan is considered fully funded if its assets exceed liabilities, Understanding Public Pension Plan's Unfunded Liability."

the 2018 total aggregated UL of $1.40T would see a reduction of $51.5B (resulting in a net UL of approximately $1.35T).

Endogeneity

Two methods were executed to rule out endogeneity assumptions. The Wald Test was performed as a joint significant test, and a lag identification approach was also used. The model was evaluated to see whether all coefficients were equal to zero. The Wald Test concludes that joint coefficients are not equal to 0 and that we can reject the null hypothesis. Table 4B shows us results that are highly statistically for all seven independent variables. Therefore, we conclude that the model has predictive power where at least one of the independent variables would help to predict our dependent variable.

A one- and two-year lagging approach was also assessed for all statistically significant variables and are shown in Table 4A. These strategies were chosen to help alleviate threats to causal identification without the need to use any other data than what is available in the data set.

Conclusion

One of the biggest financial concerns within the public and private retirement plan landscape is the ability to pay out guaranteed lifetime pension obligations for current and future employees. To determine the magnitude of the obligation, the UL (assets minus liabilities) was selected as the dependent variable and measured against several variables grouped in three categories that included contributions, economic methods, and investment strategy.

Results conclude that three of the fifteen predictor variables could help close the ULG. These variables include the discount rate, avg 10-year returns and equity allocation on an aggregated statewide plan basis. As pointed out during data analysis, other proxies such as 10-year Treasury notes, 30-year bonds or high-grade municipal bonds should be set as a standard for calculation of the assumed investment return known as the discount rate. Additionally, an investment policy statement that outlines the most optimal asset allocation strategy should be reviewed each year. Standard risk tolerance questions should be answered and documented during the plan annual review the results should help develop the right asset mix of equities and fixed income investments for the plan.

Results also reveals an increase to the UL when ARC increases by 1%. In order to reverse the increase, it is my recommendation that the ARC payment by the employer be required, not optional. This approach will ensure that full payments are made and in a timely manner. There should also be legal ramifications

levied if the employer does not pay on a timely basis. In the public sector, there may be political resistance to amend the contribution requirements. However, this may be the right season (post-Covid 19) for politics to be set aside as many public employers simply do not have enough funds to cover its liabilities. Another finding shows that, employees contributions added to the total ARC payment made by the employer, the UL begins to slightly reduce. Public plans may consider a plan design that require an employee contribution.

The private and public sectors of DB plans are faced with three main risks. The private sector established rules and regulations (ERISA) to deal with these concerns and help to mitigate these risks. However, the public sector has not addressed these concerns through a formal mandate such as ERISA. ERISA-like rules and regulations should be introduced in the public sector to address the issue of compliance. There are currently no minimum pension plan funding requirements for public plans in the U.S. and therefore the ARC payment should no longer be an optional payment. Additionally, there is currently no external insurance or pension guarantee fund for DB public plans. Insurance should be required for public plans like the Pension Benefit Guaranty Corporation (PBGC) that exists today for private DB plans which protects plan benefits up to statutory limits. A statutory lien should be enforced if an employer has not made requirement minimum contributions and unpaid amounts total more than a certain dollar threshold. Premiums should be financed by tax revenue to provide protection should the employer become unable to continue the plan. This type of benefit security is necessary as public plans seek to protect their employees (plan members). Also, there should be minimum reporting requirements for public pension plans to an entity such as PBGC that can update plan participants about insurance risks. One suggestion would be that underfunded plans (i.e., below 80% funded ratio) should be closely monitored and expected to report plan statistics more frequently than those that are adequately funded.

Being transparent is another essential area of improvement in the public sector, oftentimes employees are not informed of the status (i.e., funding status, investment strategy, etc.) of DB plans. There has been a shift as noted earlier that DB public plans are no longer solely investing in fixed income investments, but across multiple asset classes. It is important to provide employees with specifics of the plan and its inherent risks. One way is to require annual group meetings with employees to review pension plan status. An annual plan disclosure and or other plan communication materials should be supplied to all DB participants within the public sector.

Both financial measures and plan design play a critical role in finding ways to reduce the UL. Further research can be conducted on testing other proxies such as 10-year Treasury notes, 30-year bonds or high-grade municipal bonds to be used as best practice when calculating the assumed investment return.

Additionally, emphasis was placed on the testing of multiple financial determinants throughout this paper. Any suggested changes to the annual required contribution (ARC) can be considered a modification to the plan design. However, research to evaluate and determine the impact of other potential plan design changes (such as age, years of service, average salary, service credits) would be a beneficial next step for this study.

Appendix

Table 1

Please note that Table 3 in listed in the introduction session

Table 2

To compute the approximate annual benefit, multiply the average of the highest 3 years (30 months) of paid salaries in Connecticut public schools by the appropriate percentage. The percentage shown are estimated based on full-time credited teaching service. Percentage will be lower for part-time employees. The minimum qualifications for retirement are: 25 years credited service at any age (20 in CT), or 20 years credited service at age 55 (15 in CT) or 10 years of credited service at age 60 (10 in CT).[21]

Retirement Percentage Chart[21]
(Connecticut public schools)

AGE	20 YRS	21 YRS	22 YRS	23 YRS	24 YRS	25 YRS
55	28.00%	29.40%	30.80%	32.20%	33.60%	35.00%
56	30.40%	31.92%	33.44%	34.96%	36.48%	38.00%
57	32.80%	34.44%	36.08%	37.72%	39.36%	41.00%
58	35.20%	36.96%	38.72%	40.48%	42.24%	44.00%
59	37.60%	39.48%	41.36%	43.24%	45.12%	47.00%
60	40.00%	42.00%	44.00%	46.00%	48.00%	50.00%

Resign at age 60 or Older	10 YRS	15 YRS	20 YRS
Percentage	10.00%	22.50%	40.00%

Table 3

Please note that Table 3 in listed in the Empirical Model session

[21] CT official State Website, https://portal.ct.gov/TRB, CT Teacher's Retirement Board, Benefit Estimator.

Table 3A: Unfunded Liability as a function of all selected independent variables
Multiple OLS Regression
Panel Data: Cross-section Data is Fixed/Period is Fixed
Groups: Balanced 1,2,3 and Unbalanced 4 and 5

	Sample Years 2003-2018						Weighted Stats	
	Independent variables	**β**	**P**		***T-Stats***	**SE**	**F-statistic**	**R^2**
Group 1 Contribution rate (Balanced)	ARCR	-.5500	0.0602	*	[-1.8801]	0.2925	44.3990	0.7672
	ARCR_ER	2.0780	0.0000	****	[4.7552]	0.4369		
Group 2 State Financial Levers (Balanced)	TAXRPC	1.2200	0.0002	****	[3.7218]	3.2800	34.4427	0.7214
	GPERC	-6.7100	0.6737	N/A	[-0.4211]	1.5900		
	BUDGDP	1.0904	0.2967	N/A	[1.0436]	1.0448		
	BUDPC	-1.0400	0.4551	N/A	[-0.7470]	1.3900		
Group 3 Assumed Discount Rate (Balanced)	DISCR	-8.2908	0.2050	N/A	[-1.2677]	6.5398	35.6445	0.7244
Group 4 Investment Returns (Unbalanced)	PRTRN	-0.3923	0.5579	N/A	[-0.5861]	0.6693	27.7342	0.7807
	EQRTN	-0.0082	0.9034	N/A	[-0.1213]	0.0682		
	FIRTN	0.3370	0.0561	*	[1.9120]	0.1762		
	A3YR	0.0008	0.9977	N/A	[0.2999]	0.0029		
	A5YR	-0.5791	0.0669	*	[-1.8336]	0.3158		
	A10YR	-2.7340	0.0076	***	[-2.6726]	1.0229		
Group 5 Investment Allocation (Unbalanced)	EQTALL	-0.3118	0.0000	****	[-5.2479]	0.0594	33.3645	0.7230
	FIALL	0.0266	0.8234	N/A	[-0.2232]	0.1191		

****p<0.001; ***.001<p<0.01; **0.01<p<0.05, *0.05<p<0.10, level of significance of effects

Table 3A shows results for all selected independent variables (15-variable linear regression). The independent variables are shown in five groups – two groups of balanced data and three groups of unbalanced panel data. Results are shown for beta, probability, T-stats, standard errors, weighted F-statistic and R2 based on author's estimation.

Table 3B: Unfunded Liability as a function of the Optimal Independent Variables (No Lag)
Panel Data: Cross-section Data is Fixed/Period is Fixed
Groups: Balanced 1 and Unbalanced 2

	Sample Years 2003-2018						Weighted Stats	
	Independent variables	**β**	***P***		***T-Stats***	**SE**	**F-statistic**	**R^2**
Balanced Group 1	ARCR	-0.5171	0.0670	*	[-1.8285]	0.2828		
	ARCR_ER	2.0101	0.0000	****	[4.8434]	0.4150	44.2005	0.7686
	TAXRPC	1.1400	0.0001	****	[3.8274]	2.9800		
	DISCR	-3.737	0.4310	N/A	[-0.7875]	4.7453		
Unbalanced Group 2	FIRTN	0.2797	0.0549	*	[1.9211]	0.1456		
	A10YR	-2.6064	0.0007	****	[-3.3823]	0.7706	30.7049	0.7541
	EQTALL	-0.1881	0.0539	*	[-1.9294]	0.0974		

****p<0.001; ***.001<p<0.01; **0.01<p<0.05, *0.05<p<0.10, level of significance of effects

Table 3B displays panel data that represent independent variables that were statistically significant (7-variables) in both the balanced and unbalanced group. The discount rate variable, although not significant at this stage is also shown. The five groups as shown in 3A were condensed two groups displayed in Table 3B. Result shown for beta, probability, T-stats, standard errors, weighted F-statistic and R2 are based on author's estimation.

Table 3C: Unfunded Liability as a function of the Optimal Independent Variables- (No Lag)
Panel Data: Cross-section Data is Fixed/Period is Fixed
Optimization of Multiple OLS Regressions- Explained Variables
One Group: Combined Balanced and Unbalanced

	Sample Years 2003-2018						Weighted Stats	
	Independent variables	**β**	***P***		***T-Stats***	**SE**	**F-statistic**	**R^2**
One Group	ARCR	-0.5171	0.2558	N/A	[-1.1367]	0.4512		
	ARCR_ER	1.9454	0.0011	****	[3.2831]	0.5925		
	TAXRPC	2.5800	0.0004	****	[3.5326]	7.2900		
	DISCR	-14.8682	0.0044	***	[-2.8526]	5.2120	42.5630	0.832
	FIRTN	0.2536	0.0328	**	[2.1371]	0.1186		
	A10YR	-1.7051	0.0032	****	[-2.9525]	0.5775		
	EQTALL	-0.0360	0.6757	N/A	[-0.4183]	0.0860		

****p<0.001; ***.001<p<0.01; **0.01<p<0.05, *0.05<p<0.10, level of significance of effects

Table 3C combines the balanced and unbalanced variables of panel data from 3B into one group. The discount rate variable now displays a reduction in the unfunded liability as statistically significant, while ARCR (Employee and Employer annual required contribution) and EQTALL (equity allocation) are no longer significant. Results shown for beta, probability, T-stats, standard errors, weighted F-statistic and R2 are based on author's estimation.

Table 4A: Unfunded Liability as a function of the Optimal Independent Variables
(Lagged One-Year and Two -Year Balanced and Unbalanced Groups as shown in Table 3B)
Panel Data: Cross-section Data is Fixed/Period is Fixed
Groups: Balanced 1 and Unbalanced 2

Sample Years 2003-2018

	Independent variables	β	*P*		*T-Stats*	SE	Weighted Stats F-statistic	Weighted Stats R^2	Lag Year
Balanced Group 1	ARCR(-1)	-0.6638	0.0132	**	[-2.4791]	0.2677			
	ARCR(-2)	-0.7780	0.0015	***	[-3.1745]	0.2450			
	ARCR_ER(-1)	1.9974	0.0000	****	[5.2974]	0.3770			
	ARCR_ER(-2)	1.7964	0.0000	****	[5.3563]	0.3353			
							40.9220	0.7562	1
	TAXRPC(-1)	1.0100	0.0245	**	[2.2505]	4.5000	37.2817	0.7410	2
	TAXRPC(-2)	7.0900	0.0964	*	[1.6634]	4.2600			
	DISCR(-1)	-4.3797	0.2550	N/A	[-1.1386]	3.8465			
	DISCR(-2)	-3.6850	0.0105	***	[-2.5604]	1.4392			
Unbalanced Group 2	FIRTN(-1)	0.3473	0.0127	**	[2.4944]	0.1392			
	FIRTN(-2)	0.2749	0.0193	**	[2.3427]	0.1173			
	A10YR(-1)	-3.0366	0.0000	****	[-4.9121]	0.6181	32.0292	0.7960	1
	A10YR(-2)	-2.2296	0.0325	**	[-2.1400]	1.0418	31.9573	0.86063	2
	EQTALL(-1)	-0.1767	0.0570	*	[-1.9050]	0.0928			
	EQTALL(-2)	-0.2808	0.0038	***	[-2.8985]	0.0968			

****p<0.001 ;***.001<p<0.01; **0.01<p<0.05, *0.05<p<0.10, level of significance of effects

Using data as shown in Table 3B, this table displays the variable results with a one- and two-year lag. Two groups are shown in Table 4. The results show that the Employer annual contribution rate (ARCR_ER) is highly statistically significant, two-year lag on discount rate (DISCR) shows statically significant and the one- year lag on the 10 year (A10YR) is even more robust when results are lagged. Results shown for beta, probability, T-stats, standard errors, weighted F-statistic and R2 are based on author's estimation.

Table 4B: Walt Test [Joint Significance Test]- No Lag
Panel Data: Cross-section Data is Fixed/Period is Fixed
One Group: Combined Balanced and Unbalanced
Dependent Variable: Unfunded Liability as a % to Assets (RUFL)

Independent Variables	**β**	**Standard Error (SE)**	**F-statistics**	**Probability 0.0000**
ARCR	-0.5236	0.4512	14.282	****
ARCR_ER	1.9379	0.5925		****
TAXRPC	2.540	7.2900		****
DISCR	-14.3630	5.2120		****
FIRTN	0.2536	0.1186		****
A10YR	-1.7051	0.5775		****
EQTALL	-0.0361	0.0860		****

****p<0.001; ***.001<p<0.01; **0.01<p<0.05, *0.05<p<0.10, level of significance of effects

The model was evaluated to see whether all coefficients were equal to zero. Results were highly significant, and we conclude that the model has predictive power where at least one of the independent variables (X) help to predict our dependent variable (Y). Results shown for beta, probability, standard errors, weighted F-statistic and R2 are based on author's estimation.

Fig. 1

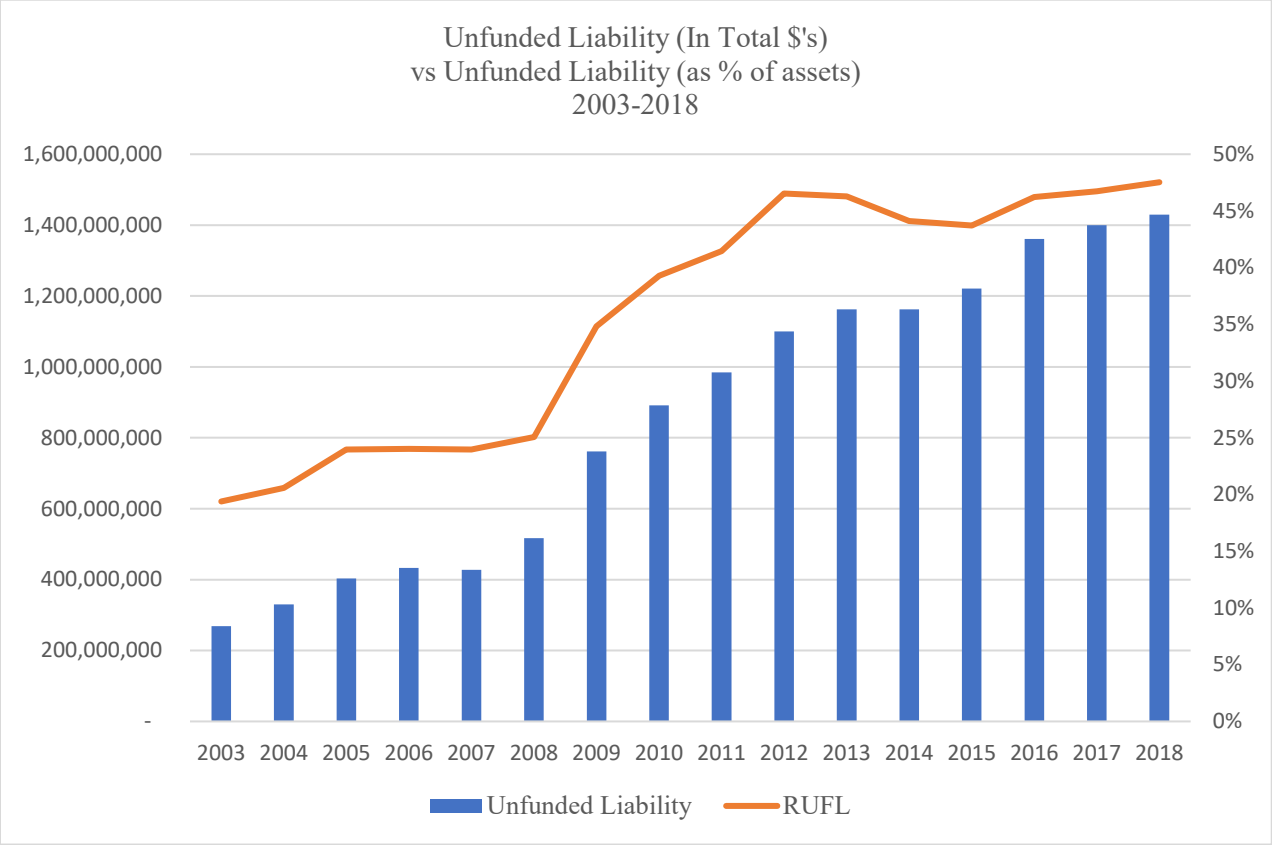

Fig. 1 displays the total aggregated unfunded liability which is the dependent variable. The total value vs. the unfunded liability as a percentage of assets from 2003-2018 is shown. The panel model was adjusted to reflect those plans that reported PPD data during 2003-2018.

Fig. 2

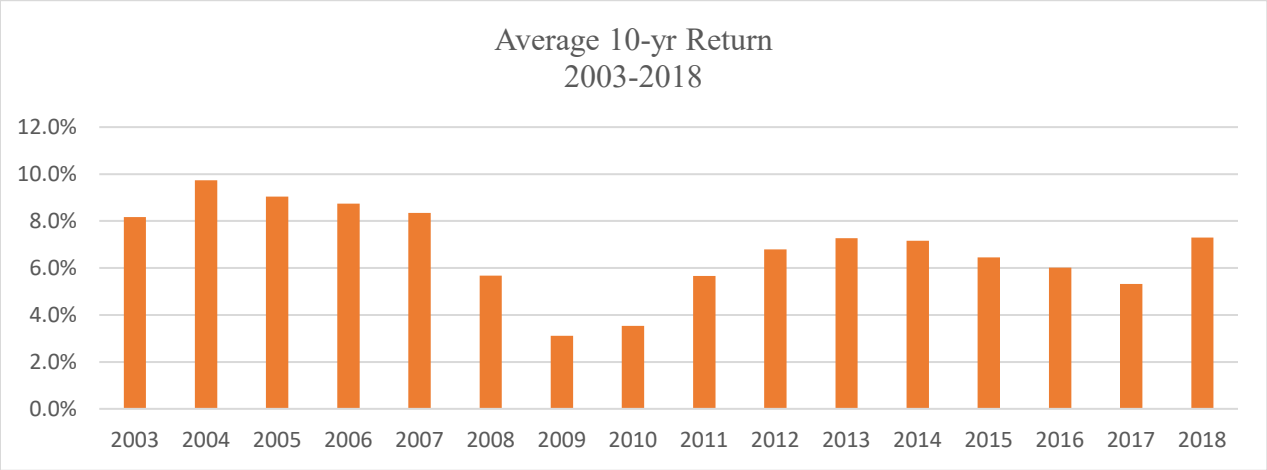

Fig. 2 shows a snapshot of the average 10-year return of all aggregated plans used in the dataset from 2003-2018. Note that 2004 shows the highest average 10-year return of 9.7% vs. lower returns 3.1% and 3.5% in 2009 and 2010.

Fig. 3

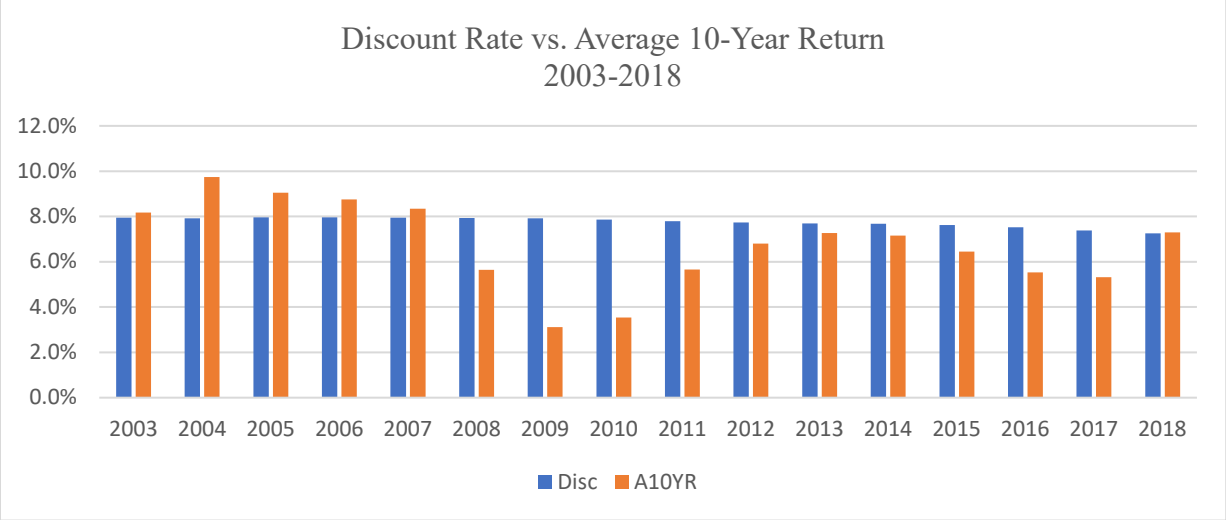

Fig. 3 shows the assumed investment return known as the discount rate vs. the actual average 10- year return. This discount rate is used to value the current cost of future pension obligations and is determined by estimating expected rates of return on all aggregated plans in dataset from 2003-2018. The average 10-year return is the return of all aggregated plans used in the dataset from 2003-2018. The discount rate shows a consistent average of 7.8% vs. the average10-year return of 6.8%.

Fig. 4

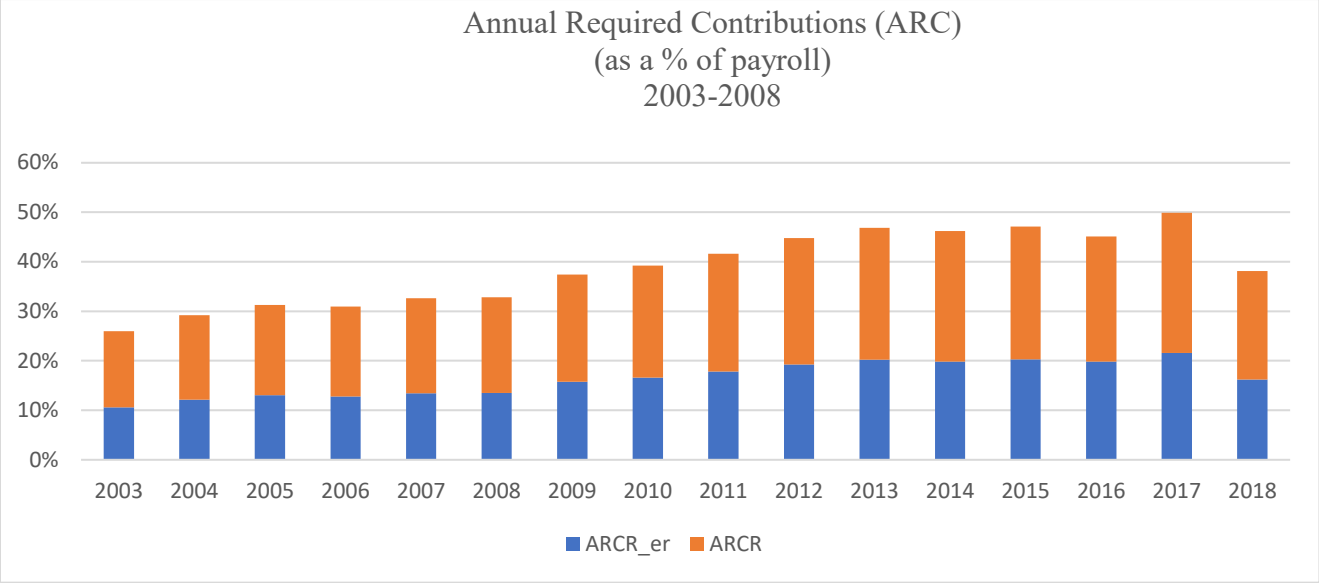

Fig. 4 shows the Annual Required Contribution by the employer [ARCR_ER] and the Annual Required Contribution by both the employee and the employer [ARCR] from 2003-2018. Note that 2003 shows the lowest total contributions of 26% of payroll and 2017 shows the highest at 50% of payroll. These numbers are shown as a percentage of payroll of all aggregated plans used in the PPD dataset from 2003-2018.

Fig. 5

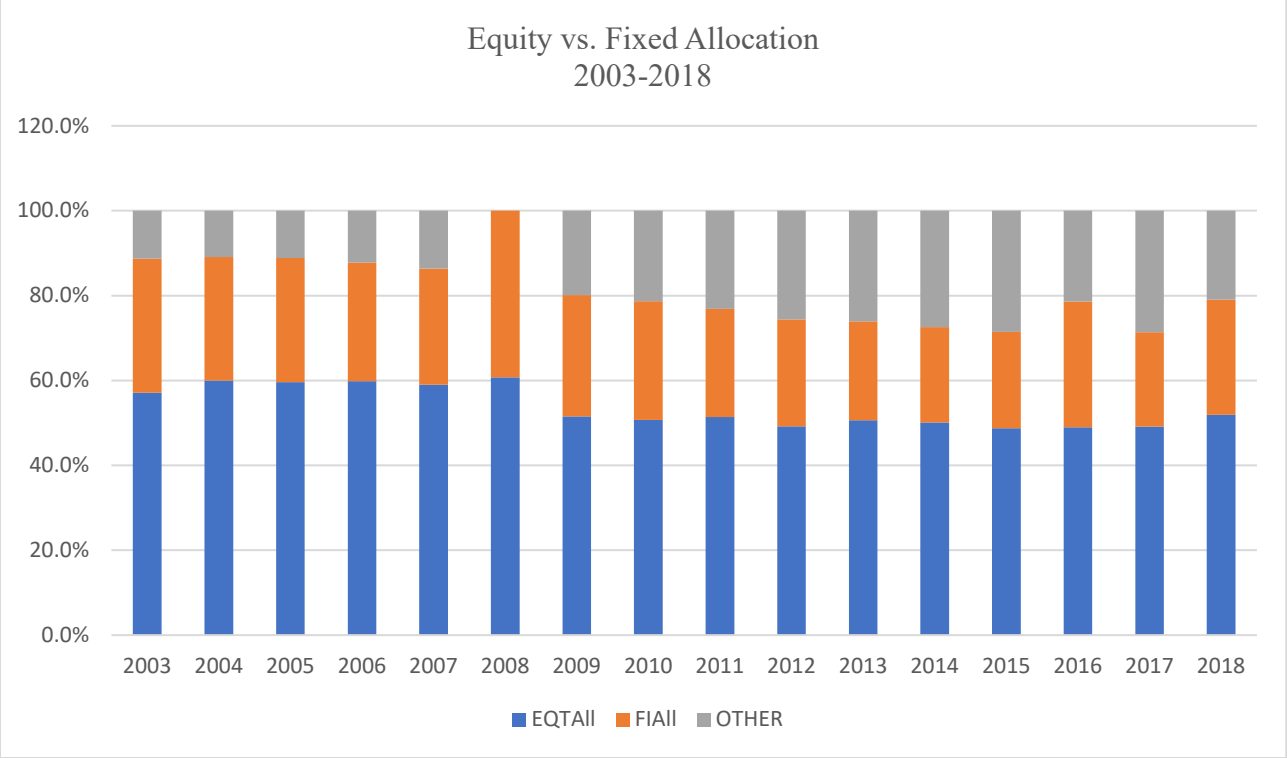

Fig. 5 shows the equity, fixed and other allocations (real estate, alternatives, private equities, commodities, cash, and other asset classes) aggregated by all plans used in dataset over the 16-year period. Note that there is higher percentage of equity allocation to fixed income allocation over each of the 16 years. Excluding 2008, assets were also allocated to real estate, alternatives, private equities, commodities, and cash. The 2008 allocation shows that the financial crisis has impacted the allocation choices of many of the public pension plans across the U.S. Overall, the chart shows that there has been a shift from fixed income to equities within public pension plans.

Fig. 6

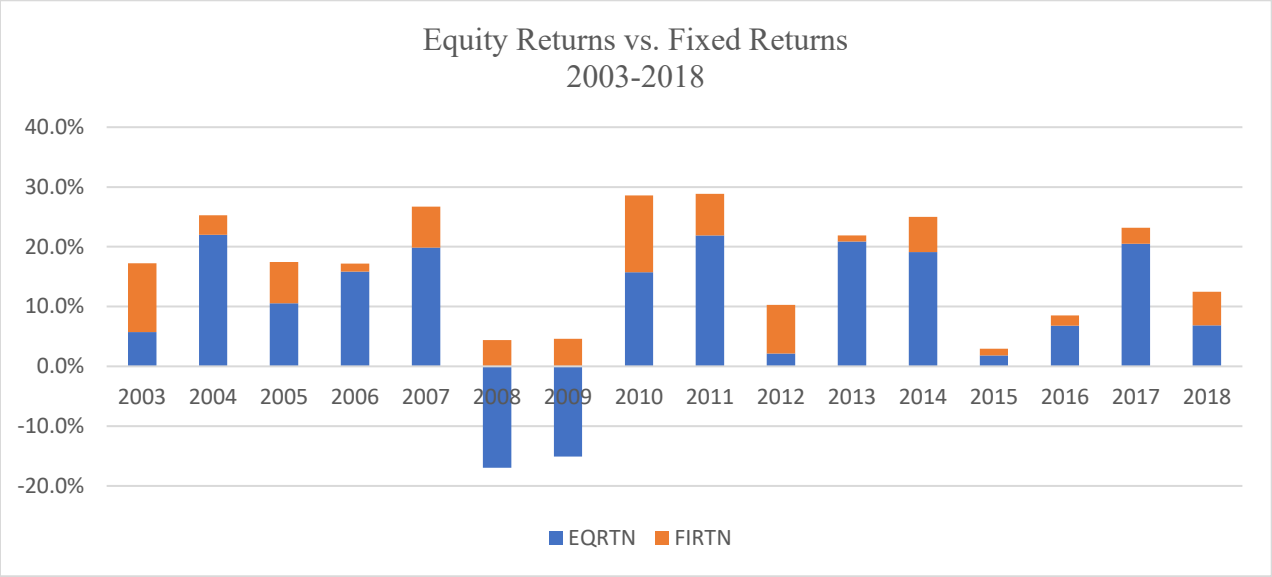

Fig. 6 shows both equity and fixed returns aggregated by all plans used in PPD dataset from 2003-2018. Note that the 2008 allocation to equities and fixed income as shown on Fig. 5 is 60.8% and 39.3% respectively and generated a negative return of 17.0% as shown in Fig 6. Additionally, the 2009 allocation to equities and fixed income as shown on Fig. 5 is 51.5% and 28.5% respectively and generated a negative return of 15.1% as shown in Fig 6.

List of U.S. Public Plans included in analysis[1]

Plan ID	State ABBR	Plan Name	Plan Full Name
1	AL	Alabama ERS	Employees' Retirement System of Alabama
2	AL	Alabama Teachers	Teachers' Retirement System of Alabama
3	AK	Alaska PERS	State of Alaska Public Employees' Retirement System
4	AK	Alaska Teachers	State of Alaska Teachers' Retirement System
5	AZ	Arizona Public Safety	Arizona Public Safety Personnel Retirement System
6	AZ	Arizona SRS	Arizona State Retirement System
94	AZ	Phoenix ERS	Phoenix Employees' Retirement System
127	AZ	Arizona State Corrections Officers	Arizona State Corrections Officers Retirement Plan
176	AZ	Tucson Supplemental RS	Tucson Supplemental Retirement System
7	AR	Arkansas PERS	Arkansas Public Employees Retirement System
8	AR	Arkansas Teachers	Arkansas Teacher Retirement System
9	CA	California PERF	California Public Employees Retirement Fund
10	CA	California Teachers	California State Teachers' Retirement System
18	CA	Contra Costa County	Contra Costa County Employees' Retirement Association
43	CA	LA County ERS	Los Angeles County Employees Retirement Association
97	CA	San Diego County	San Diego County Employees Retirement Association
98	CA	San Francisco City & County	San Francisco City & County Employees' Retirement System
111	CA	University of California	University of California Retirement Plan
137	CA	Alameda County ERS	Alameda County Employees' Retirement Association
138	CA	Kern County ERS	Kern County Employees' Retirement Association
139	CA	Los Angeles ERS	Los Angeles City Employees' Retirement System
140	CA	Los Angeles Fire and Police	Los Angeles City Fire and Police Pension System
141	CA	Los Angeles Water and Power	Los Angeles Water and Power Employees' Retirement Plan
142	CA	Orange County ERS	Orange County Employees Retirement System
143	CA	Sacramento County ERS	Sacramento County Employees' Retirement System
144	CA	San Diego City ERS	San Diego City Employees' Retirement System
13	CO	Colorado Municipal	Colorado Public Employee Retirement Association-Local Division
14	CO	Colorado School	Colorado Public Employee Retirement Association-School Division
15	CO	Colorado State	Colorado Public Employee Retirement Association-State Division
22	CO	Denver Employees	Denver Employees Retirement Plan
23	CO	Denver Schools	Colorado Public Employee Retirement Association-Denver Public Schools Division
16	CT	Connecticut SERS	Connecticut State Employees Retirement System
17	CT	Connecticut Teachers	Connecticut State Teachers' Retirement System
128	CT	Connecticut Municipal	Connecticut Municipal Employees Retirement System
21	DE	Delaware State Employees	Delaware State Employees' Pension Plan
195	DE	Delaware County and Municipal Employees	Delaware County and Municipal Other Employees

List of U.S. Public Plans included in analysis (ctd)[1]

Plan ID	State ABBR	Plan Name	Plan Full Name
	FL	Florida RS	Florida Retirement System
27	GA	Georgia ERS	Employees' Retirement System of Georgia
28	GA	Georgia Teachers	Teachers Retirement System of Georgia
161	GA	Atlanta ERS	Atlanta General Employees' Pension Fund
29	HI	Hawaii ERS	Employees' Retirement System of the State of Hawaii
31	ID	Idaho PERS	Public Employee Retirement System of Idaho
11	IL	Chicago Teachers	Public School Teachers' Pension and Retirement Fund of Chicago
32	IL	Illinois Municipal	Illinois Municipal Retirement Fund
33	IL	Illinois SERS	State Employees' Retirement System of Illinois
34	IL	Illinois Teachers	Teachers' Retirement System of The State of Illinois
35	IL	Illinois Universities	State Universities Retirement System of Illinois
145	IL	Chicago Municipal	Chicago Municipal Employees' Annuity Benefit Fund
36	IN	Indiana PERF	State of Indiana Public Employees' Retirement Fund
37	IN	Indiana Teachers	Indiana State Teachers' Retirement Fund
38	IA	Iowa PERS	Iowa Public Employees' Retirement System
129	IA	Iowa Municipal Fire and Police	Municipal Fire and Police Retirement System of Iowa
39	KS	Kansas PERS	Kansas Public Employees Retirement System
179	KS	Wichita ERS	Wichita Employees' Retirement System
40	KY	Kentucky County	County Employees Retirement System of Kentucky
41	KY	Kentucky ERS	Kentucky Employees Retirement System
42	KY	Kentucky Teachers	Teachers' Retirement System of the State of Kentucky
44	LA	Louisiana SERS	Louisiana State Employees' Retirement System
45	LA	Louisiana Teachers	Louisiana State Teachers Retirement System
163	LA	Baton Rouge City Parish RS	Baton Rouge City Parish Employees' Retirement System
197	LA	Louisiana Municipal Employees	Louisiana Municipal Employees
46	ME	Maine Local	Maine Public Employees Retirement System - Participating Local Districts
47	ME	Maine State and Teacher	Maine Public Employees Retirement System - State and Teacher Retirement Program
48	MD	Maryland PERS	Maryland State Retirement and Pension System - Employees Combined System
49	MD	Maryland Teachers	Maryland State Retirement and Pension System - Teachers Combined System
155	MD	Baltimore Fire and Police	Baltimore Fire and Police Employees' Retirement System
50	MA	Massachusetts SRS	Massachusetts State Retirement System
51	MA	Massachusetts Teachers	Massachusetts Teachers' Retirement System
52	MI	Michigan Municipal	Municipal Employees' Retirement System of Michigan
53	MI	Michigan Public Schools	Michigan Public School Employees' Retirement System
54	MI	Michigan SERS	Michigan State Employees' Retirement System
57	MN	Minnesota State Employees	Minnesota State Employees Retirement Fund
58	MN	Minnesota Teachers	Teachers Retirement Association of Minnesota

List of U.S. Public Plans included in analysis[1]

Plan ID	State ABBR	Plan Name	Plan Full Name
103	MN	St. Paul Teachers	St. Paul Teachers Retirement Fund
133	MN	Minnesota Police and Fire	Minnesota Public Employees Police & Fire Plan
59	MS	Mississippi PERS	Public Employees' Retirement System of Mississippi
60	MO	Missouri DOT and Highway	Missouri Department of Transportation and Highway Patrol Employees' Retirement Syst
61	MO	Missouri Local	Missouri Local Government Employees Retirement System
62	MO	Missouri PEERS	Public Education Employee Retirement System of Missouri
63	MO	Missouri State Employees	Missouri State Employees' Retirement System
64	MO	Missouri Teachers	Public School Retirement System of Missouri
102	MO	St. Louis School Employees	Public School Retirement System of the City of St. Louis
65	MT	Montana PERS	Montana Public Employees' Retirement System
66	MT	Montana Teachers	Teachers' Retirement System of Montana
162	NE	Omaha School	Omaha School Employees' Retirement System
203	NE	Omaha ERS	Omaha City Employees Retirement System
68	NV	Nevada Police Officer and Firefighter	Public Employees' Retirement System of Nevada - Police and Firefighters Plan
69	NV	Nevada Regular Employees	Public Employees' Retirement System of Nevada - Regular Employees Plan
70	NH	New Hampshire RS	New Hampshire Retirement System
190	NH	Manchester Employees' Contributory Retirement System	Manchester Employees' Contributory Retirement System
71	NJ	New Jersey PERS	Public Employees' Retirement System of New Jersey
72	NJ	New Jersey Police & Fire	The Police and Firemen's Retirement System of New Jersey
73	NJ	New Jersey Teachers	Teachers' Pension and Annuity Fund of New Jersey
202	NJ	Jersey City Municipal Employees Pension Fund	Jersey City Municipal Employees Pension Fund
74	NM	New Mexico PERA	Public Employees Retirement Association of New Mexico
75	NM	New Mexico Educational	Educational Retirement Board of New Mexico
76	NY	New York City ERS	New York City Employees' Retirement System
77	NY	New York City Teachers	Teachers' Retirement System of the City of New York
78	NY	New York State Teachers	New York State Teachers' Retirement System
83	NY	NY State & Local ERS	New York State and Local Retirement System - Employees' Retirement System
84	NY	NY State & Local Police & Fire	New York State and Local Retirement System - Police and Fire Retirement System
79	NC	North Carolina Local Government	North Carolina Local Governmental Employees' Retirement System
80	NC	North Carolina Teachers and State Employees	Teachers' and State Employees' Retirement System of North Carolina
182	NC	Charlotte Firefighters' RS	Charlotte Firefighters' Retirement System
81	ND	North Dakota PERS	North Dakota Public Employees Retirement System
82	ND	North Dakota Teachers	North Dakota Teachers' Fund for Retirement

List of U.S. Public Plans included in analysis[1]

Plan ID	State ABBR	Plan Name	Plan Full Name
85	OH	Ohio PERS	Ohio Public Employees Retirement System
86	OH	Ohio Police & Fire	Ohio Police & Fire Pension Fund
87	OH	Ohio School Employees	School Employees' Retirement System of Ohio
88	OH	Ohio Teachers	School Employees' Retirement System of Ohio
160	OH	Cincinnati ERS	Cincinnati Employees' Retirement System
89	OK	Oklahoma PERS	Oklahoma Public Employees Retirement System
90	OK	Oklahoma Teachers	Teachers' Retirement System of Oklahoma
134	OK	Oklahoma Police	Oklahoma Police Pension and Retirement System
91	OR	Oregon PERS	Oregon Public Employees Retirement System
92	PA	Pennsylvania School Employees	Public School Employees' Retirement System of Pennsylvania
93	PA	Pennsylvania State ERS	Pennsylvania State Employees' Retirement System
136	PA	Pennsylvania Municipal	Pennsylvania Municipal Retirement System
152	PA	Philadelphia Municipal	Philadelphia Municipal Retirement System
95	RI	Rhode Island ERS	Employees' Retirement System of Rhode Island
96	RI	Rhode Island Municipal	Rhode Island Municipal Employees' Retirement System
99	SC	South Carolina Police	South Carolina Police Officers Retirement System
100	SC	South Carolina RS	South Carolina Retirement System
101	SD	South Dakota RS	South Dakota Retirement System
109	TN	TN Political Subdivisions	Tennessee Political Subdivisions Retirement Plan
110	TN	TN State and Teachers	Tennessee State and Teachers' Retirement Plan
158	TN	Nashville-Davidson ERS	Nashville Davidson Metropolitan Employee Benefit System
12	TX	Austin ERS	City of Austin Employees' Retirement System
104	TX	Texas County & District	Texas County & District Retirement System
105	TX	Texas ERS	Employees Retirement System of Texas
106	TX	Texas LECOS	Texas Law Enforcement and Custodial Officer Supplemental Retirement Fund
107	TX	Texas Municipal	Texas Municipal Retirement System
108	TX	Texas Teachers	Teacher Retirement System of Texas
201	TX	Dallas ERS	Dallas Employees Retirement System
204	TX	Houston Municipal	Houston Municipal Employees Retirement Fund
112	UT	Utah Noncontributory	Utah Public Employees Noncontributory Retirement System
135	UT	Utah Public Safety	Utah Public Safety and Firefighter Retirement Plan
113	VT	Vermont State Employees	Vermont State Employees' Retirement System
114	VT	Vermont Teachers	State Teachers' Retirement System of Vermont
199	VT	Vermont Municipal Employees	Vermont Municipal Employees
25	VA	Fairfax County Schools	Educational Employees' Supplementary Retirement System of Fairfax County
115	VA	Virginia RS	Virginia Retirement System
117	WA	Washington LEOFF Plan 2	Washington Law Enforcement Officers and Firefighters Plan 2
119	WA	Washington PERS 2/3	Washington Public Employees' Retirement System
120	WA	Washington School Employees	Washington School Employees Plan 2

List of U.S. Public Plans included in analysis[1]

Plan ID	State ABBR	Plan Name	Plan Full Name
122	WA	Washington Teachers Plan 2/3	Washington Teachers Plan 2/3
123	WV	West Virginia PERS	West Virginia Public Employees' Retirement System
124	WV	West Virginia Teachers	West Virginia Teachers' Retirement System
125	WI	Wisconsin RS	Wisconsin Retirement System
151	WI	Milwaukee City ERS	Milwaukee City Employees' Retirement System
126	WY	Wyoming Public Employees	State of Wyoming Retirement System

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