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Education of Mathematically Talented Students in Hungary

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Hungary is famous for its production of large numbers of highly talented mathematicians and physicists. This study explores the Hungarian system for educating mathematically talented secondary school students with the goal of identifying successful features that may be applicable to education in the United States. Highlights of the Hungarian approach include an emphasis on problem solving, problem posing, detailed explanation or proof for solutions, and development of mathematical creativity through the search for multiple solution paths.

Introduction

Pál Erdős, Lipót Fejér, László Lovász, John von Neumann, George Pólya, Lajos Pósa, Eugene Wigner—are just a few of the many notable Hungarian mathematicians of the 20th century. In fact, Hungary has been cited as producing the “largest per capita number of mathematicians and physicists during the first half of the 20th century” (Vogeli, 1997, p. 11). One of the hallmarks of the Hungarian mathematics education system is the formation of special mathematics schools for exceptionally talented students. The origin of similar mathematics and science magnet schools in the United States and Russia has been traced back to the first special schools for mathematics in Budapest that were founded in the early part of the 20th century (Vogeli, 1997; Wieschenberg, 1984). Visits to schools such as Fazekas Gimnazium in Budapest reveal that these special schools are models of teaching mathematically gifted students in an environment focused on enrichment, creative problem solving, and rigorous mathematical discussion. Hungary’s success on TIMSS, PISA, and other IEA studies has highlighted the value of learning from their mathematics education system:

From a participation perspective, the Hungarians have it both ways. Not only do they provide advanced mathematical experience to a large percentage of the cohort, and thereby increase dramatically the sum of mathematical knowledge in the culture, but they also do it without sacrificing the talents of their most capable students. As a model for both providing opportunity and creating a pool of talent, Hungary’s bears scrutiny. (Kifer, 1989, p. 69)

Methodology

This paper summarizes reflections from a larger study (Connelly, 2010) based on data collected from primary source documents and in-depth interviews with current leaders in the Hungarian mathematics education system.

Historical background information was gathered from ministry of education publications, mathematical and pedagogical professional journals, and earlier dissertations in the field. Examining textbooks and school entrance and leaving exams helped identify changes in the nature and level of mathematics and standards expected of Hungarian secondary school students. Individual school, camp, competition, and journal websites also provided information about their programs and offerings for mathematically talented students.

Interview subjects (referred to here as respondents A-H) were chosen based on their years of experience, knowledge of the system for mathematically talented students, and level of involvement with shaping these programs for the future. Interview participants included secondary school teachers from some of the special mathematics high schools in Budapest (many of whom had also been students in a special mathematics class themselves), as well as some mathematicians and professors who were members of the very first special mathematics class in Hungary. In addition to the eight full interviews conducted, personal communications took place with thirteen other teachers, mathematicians, and graduate students throughout the research process. These individuals will be referred to as respondents I-U. It is hoped that by combining a variety of objective sources such as exams and textbooks with the personal commentary from the interviews, the study will be able to paint a more full picture of the Hungarian mathematics education system for gifted students and how it compares to prevailing trends in the United States.

Giftedness in Cultural Context

In general, some form of special education for talented students is something that Hungary and the United States have in common while few other dimensions are similar. It is also a relatively unique dimension—many countries have no provisions for educating gifted students. Often these differences stem from a country’s political and cultural history or from an underlying difference in the

conception of giftedness (Hernández de Hahn, 2000; Phillipson & McCann, 2007). In the United States and Canada, for example, it is commonly believed that differences in student achievement are the result of “natural endowments,” whereas in many Asian countries the common belief is that differences in ability are the result of hard work and effort (Moon & Rosselli, 2000).

Conceptions of giftedness and the designs of gifted education programs also are strongly influenced by local political processes. In a democratic political system such as in the United States, there are alternating pushes toward excellence and egalitarianism (Tannenbaum, 2000). On the one hand, we pride ourselves on being a “land of opportunity,” where anyone can “pull themselves up by their bootstraps” and achieve success regardless of race, gender, or socio-economic status. This point of view rewards those with exceptional talent and seeks to provide them with opportunities to maximize that talent. On the other hand, the country was founded on the fundamental principle that “all men are created equal,” and our origins are as a nation that overthrew a system of government with a ruling aristocratic class. We have been wary of allowing a new elite class to develop (Gallagher, 1979). In terms of gifted education, this means that during “placid” times, gifted programs are viewed as “undemocratic,” elitist, and unfair allocations of resources (Gallagher, 1979, p. 3). During times like the Sputnik era, on the other hand, there has been a marked increase in public and governmental support for gifted education, specifically in science and mathematics. At that point in time, mathematically talented students came to be viewed as a national resource, and their appropriate and successful education became a national responsibility.

On the surface, it seems that socialism would call for equal education for all students—no special provisions, no developing an “intelligentsia” that is separate from the working class. However, according to Swetz (1978), the development of separate programs for talented students was actually a very common phenomenon; most socialist countries appear to have embraced the concept of giftedness as a national resource and the idea that talented individuals should be encouraged in their interests and given a strong educational foundation so that they can go on to use their talents for the good of their country. As explained by a Hungarian Ministry of Education official in 1968, “it is an important social and personal interest to educate pupils who have a special inclination to a subject or a branch of sciences. It is an important task of the socialist pedagogy and school policy to educate highly talented pupils. This task is served by the specialized classes” (Buti, 1968, p. 151). This mentality echoes the U.S. educational system’s response to Sputnik and the need for qualified engineers and scientists in order for the U.S. to be competitive in the space race during the Cold War. Indeed, much of the increased development of gifted programs in the post-Sputnik era was focused specifically

on improving mathematics and science training for talented students (Tannenbaum, 2000). A similar trend developed in Hungary, placing a particular emphasis on developing talent in mathematics as a national resource, and success in international mathematics competitions became a source of national pride.

Special Mathematics Secondary Schools

While competitions and extracurricular activities for mathematically talented students had existed in Hungary since the turn of the 20th century, the advent of the Cold War led to the development of a new type of program *within* the school system: specialized tracks in mathematics at select secondary schools around the country. The first such class was founded at Fazekas Gimnazium in Budapest in 1962, and there are now 11 such schools around the country. At each of the four special mathematics schools in Budapest, the special mathematics class currently consists of approximately thirty students, typically split into two groups. At some schools, these groups are divided arbitrarily (e.g., alphabetically by last name), but at others they are divided by ability: “strong and even stronger” (A, personal communication, 2009). Unlike the high school system in the United States, where students have a different teacher in a different mathematical subject each year (e.g., Algebra in 9th grade, Geometry in 10th grade, Algebra II/Trigonometry in 11th grade, and PreCalculus in 12th grade), in Hungary the mathematics teachers stay with the same group of students throughout their secondary school career.¹ Each class has a pair of mathematics teachers, one of whom is the head teacher for that class, teaching 65-80% of the mathematics lessons each week. The other teacher is then responsible for the remaining two or three lessons. The two teachers work together to determine pacing and divide up the topics each will cover (A, E, I, personal communication, 2009).

Each of the schools offering a special mathematics class has other tracks as well. These can include specialization in humanities, foreign languages, natural sciences, chemistry, or a general track. In comparison, most of the specialized mathematics and science magnet schools in the U.S. are in fact entire schools, not just single classes—see Applbaum (1958), Gallagher (1979), Green (1993), and Vogeli (1997) for more detailed description of the various American schools and their historical development. In these schools, students typically have the opportunity to take mathematics and science courses that are more advanced or outside the standard secondary school curriculum, but may not be required to follow any

¹ Now that the special mathematics classes are offered from grade 7 to grade 12, the students typically have one pair of teachers for the first two years, and another pair of teachers for grades 9-12.

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particular track. While the U.S. magnet school system may serve a greater proportion of students than the Hungarian special mathematics classes, the level of specialization in a single subject does not appear to be as high.

Interestingly, in Hungary there is no distinction between separate mathematical subjects—each year, students are enrolled in “Mathematics” and the teacher has control over when and how various topics are covered. This is very different from the traditional U.S. mathematics track, where students step through a sequence of distinct courses. One interviewee explained the benefits of the Hungarian approach in developing a richer understanding of mathematics as a whole:

We have no [separate] courses—mathematics is mathematics. Just one mathematics. For me it would be... of course I can imagine myself like in the university I teach Algebra only... but here especially in high school, you are so free to put things together and show students the relations between certain aspects of mathematics. It’s a false image of mathematics, a working mathematician is of course a specialist working in a field, but as a cultural heritage you cannot split mathematics into sequential, distinct courses. (*E*, personal communication, 2009)

The fluidity of topics combined with the fact that a teacher stays with a single group of students for at least four years means that teachers have the ability to determine the structure of the mathematics curricula within those four years. As one teacher explained, “you can design the arc, how you build mathematics. That’s beautiful. It gives you a lot of freedom, and that’s very good” (*A*, personal communication, 2009).

In keeping with the traditional model, current special mathematics classes continue to cover material beyond the normal curriculum and cover standard topics more in depth, rather than speeding ahead to college-level mathematics. As László Lovász recalls from his own and his son’s experiences in the special mathematics class at Fazekas as compared to their time in the United States, “[in Hungary] there was no great pressure to run ahead, and try to accomplish, say, half the undergraduate curriculum; that was not the goal. The goal was to learn what mathematics was about and to become good in problem solving” (in Webster, 2008). According to interviewed teachers, one of their pedagogical priorities is to help students feel the excitement of discovery in mathematics (*E*, *H*, personal communication, 2009). This is often accomplished through the use of interesting and challenging problems as a way to explore the material, rather than a standard lecture format.

Special math students could just be mesmerized by giving them problems and actually getting together questions that are so far unanswered. It’s really inspiring for them. That’s the point when

they really understand that mathematics is an open discipline. And they realize that even with basic knowledge and basic tools you can get to a point in mathematics where you are actually the creator. According to my experience, this is one of the most inspiring tools. (*C*, personal communication, 2009)

Although some textbooks have been published for the special mathematics classes, they appear to be almost never used, and the small size of the prospective audience limits prospects for new editions. Rather than being organized by grade-level as standard textbooks are written, the special mathematics books are a collection of small volumes on particular fields of mathematics, including Analysis I, Analysis II, Computer Science, Geometry, Logic, Vectors and Coordinate Geometry. They do not cover the entire curriculum, serving more as supplemental or optional material (*A*, personal communication, 2009). More commonly, teachers use an “exercise book”—a collection of problems, with no topical explanations or descriptions—and also incorporate problems gathered from past competitions, mathematical journals, or other Hungarian problem books. There is a long tradition in Hungary of teaching problem solving (Pólya, 1988) and problem posing, both in competitions and in the classroom. Multiple teachers interviewed described their pedagogical approach as primarily consisting of problem selection; in particular, crafting sequences of problems that lead students through the subject step-by-step. Lajos Pósa, member of the first special mathematics class and known throughout Hungary for his development of a series of camps for talented students, described how he selects problems when working with different types of students:

There is also a big difference in how to bring the discovery approach to different types of students, such as in how to choose the problems. The method involves building a staircase to the goal, and with regular kids you need smaller, more frequent steps whereas with talented students the steps can be a little further apart/steeper. (Pósa, personal communication, 2009)

As another teacher explained, his goal in problem posing is to “make them think. And then let them think” (*A*, personal communication, 2009). The additional hours per week that students spend on mathematics allow for this level of in-depth exploration and discussion of problems, while still covering the standard, required curricular material. Students spend time working through a problem individually or in small groups, and then discuss the results as a class. The emphasis on problem solving also has led to an increased focus on proof writing, as students are required to explain the reasoning behind their own discoveries rather than having the result taught to them directly (*H*, personal communication, 2009).

Assessment consists of a combination of written and oral exams. During class discussions students frequently are prompted to explain their reasoning or are asked to come to the board and write out their steps for the rest of the class to see (*I*, personal communication, 2009). In many of the classes observed, the teacher repeatedly asked, “Did anyone get a different solution?” or “Did anyone solve it a different way?” rather than stopping after one solution, or only showing the standard algorithm. This approach emphasizes the connections between different areas of mathematics and encourages the development of mathematical creativity. Mathematical creativity is further encouraged by the rewarding of “clever failures,” as Wiczerkowski et al. (2000) suggested—even students who make a mistake in their solution are often congratulated for the progress they did make and for how their approach contributes to the richness of the whole class’s discussion (*E*, personal communication, 2009). Other students will then be asked to find a counterexample or a flaw in the proof, or if multiple solutions were presented, the class may discuss which method they prefer and what are the mathematical merits of each approach. Another interviewee encourages students who have already solved the problem to give a mathematical hint to the rest of the class, or to ask “the next question” and attempt to generalize their solution (*H*, personal communication, 2009). This emphasis on problem solving, discussing multiple paths to a solution, and problem posing reinforces Hungary’s long-standing tradition of mathematics competitions.

Extracurricular Programs

One of the most famous mathematics competitions in Hungary, the Eötvös Competition, is considered “the first mathematical Olympiad of the modern world” (Koichu & Andzans, 2009, p. 287). Founded in 1894, it was designed for students who had just graduated from secondary school. The competition consisted of three questions based on the mathematics of the secondary school curriculum. The competition was designed to test problem-solving ability and mathematical creativity more than sheer knowledge. As one winner of the prize explained, “the problems are selected, however, in such a way that practically nothing, save one’s own brains, can be of any help ... the prize is not intended for the good boy; it is intended for the future creative mathematician” (Rado, as quoted in Wieschenberg, 1984, p. 32). Again, the emphasis is more on creativity and explanation of one’s reasoning than it is on demonstration of factual knowledge the way many assessments in the United States are set up.

The prominence of the Eötvös competition soon led to the development of a wide variety of supporting activities designed to help prepare students for the competition, including the publication of new types of problems each

month in the journal *KöMaL*, the initiation of *KöMaL*’s own year-long competition, and the formation of after-school “study circles” for interested students to spend additional time working on problems and practicing for the competition. Later, more competitions were founded at the school, local, regional, national, and international levels, with local competitions often serving as “feeders” into the larger, nation-wide competitions. When the first special mathematics class was founded in 1962, students were invited to the class on the basis of their results in a local Budapest competition. In this sense, the traditional Hungarian system for educating mathematically talented students could be considered “competition-driven.” Competitions were used to determine the input to the system; they drove the development of the content of the system both in the school curriculum and in supporting extracurricular activities. They were used to measure the output of the system and to rank students, teachers, and schools according to their competition results. The Hungarian approach served as a model for many other countries in Eastern and Central Europe, the former Soviet Union, and the United States (Koichu & Andzans, 2009, p. 289).

One competition is considered in a separate category from all the rest. It emphasizes careful thinking and stamina over speed, lasting an entire school year rather than a few hours. This is the competition run by the journal *KöMaL*, which publishes problems each month to which students submit solutions and accrue points over the course of the year. The *KöMaL* journal and competition have played a significant role in the development of mathematically talented students in Hungary over the past century, in no small part because of the prominence of previous winners:

This can be stated for sure: Almost everyone who became a famous or nearly famous mathematician in Hungary, when he or she was a student, they took part in this contest. I actually personally do not know anyone among them who would be a counterexample to that statement. (Peter Hermann, *KöMaL* editor, in Webster, 2008)

As Csapo (1991) pointed out in *Math Achievement in Cultural Context: The Case of Hungary*, the expectation of success based on previous success has created a kind of self-fulfilling prophecy in Hungarian mathematics education. In other words, the tradition of excellence breeds excellence. In the online introduction for *C2K: Century 2 of KöMaL* (1999), one of *KöMaL*’s special English-language issues, the editor shares a particularly appropriate story about the value of tradition:

There is a joke about an American visitor, who, wondering about the fabulous lawn of an English mason, asks the gardener about the secret of this miracle. The gardener modestly reveals that all

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that has to be done is daily sprinkling and mowing once a week.

– So very simple?

– Yes. And after four hundred years you may have this grass. (Berzsenyi)

In fact, the story of the English gardener reflects not just the more than 100-year heritage of KöMaL, but also the value of engaging in mathematics on a regular, sustained basis, which is one of the key features of the KöMaL competition. The long duration and continuous effort required make the KöMaL competition quite different from other national and international competitions, and also make it one of the cornerstones of Hungary's approach to encouraging and identifying mathematically talented students.

Conclusions

A tradition of excellence in mathematics in Hungary may be one of the driving forces behind Hungarian students' continued success today, but it is not the only factor. Hungary also presents a good example of the impact individual mathematicians and educators can have on entire generations of future mathematicians, from Lipot Fejér in the beginning of the 20th century to Lajos Pósa in the past two decades. This individualized aspect of the Hungarian system could be hard to duplicate in a system as widespread as that in the United States. It may be possible, however, to introduce some of the Hungarian innovations on a smaller scale. One school district or state may be able to reproduce an environment very much like that in Hungary, since they have local control over establishment of special schools, choice of curricula, etc.

Although Hungarian teachers cite cultural tradition rather than educational theory as the motivation and basis for their approach, the special mathematics classes and wide range of extracurricular activities provide numerous examples of problem posing, as suggested by Kilpatrick (1987) and Silver (1997), and support the mathematical creativity research conducted by Sriraman (2008a, 2008b). Just as Sriraman (2008a) suggested, students are able to develop their mathematical creativity when they are given “non-routine problems with complexity and structure, that require not only motivation and persistence but also considerable reflection” (p. 27). Further analysis of the problem sequences designed by some of Hungary's top mathematics teachers could help introduce American teachers to this discovery-based approach. Efforts are ongoing to translate more of the problems into English to make them a more accessible resource for teachers and students around the world.

All of these pedagogical techniques reflect the Hungarian tradition of teaching students how to “think like a mathematician” rather than just perform calculations,

epitomizing the distinction Wiczerkowski et al. (2000) made between “qualitative” versus “quantitative” approaches to mathematics education. This “qualitative” approach is also reflected in the structure of exams and competitions—in Hungary, multiple choice exams are extremely rare, and detailed proofs or written explanations are required instead. An emphasis on problem solving and development of mathematical creativity as opposed to just acceleration through the standard curriculum are hallmarks of the Hungarian mathematics education system for talented students. This likely contributes to Hungary's output of so many productive mathematicians and suggests a model that programs for talented students in the United States could follow as well.

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