5-2003

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Recommended Citation

Marsh, R. F., & Nam, S-H. (2003). The impact of increasing user expectations on machine replacement. *IIE Transactions, 35*(5), 457-466. Doi: 10.1080/07408170390187906

IIE Transactions (2003) **35**, 457–466
0740-817X © 2003 "IIE"
0740-817X/03 $12.00+.00
DOI: 10.1080/07408170390187906

The impact of increasing user expectations on machine replacement

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Received November 1999 and accepted March 2002

This study explores the traditional equipment maintain or replace decision under scenarios of increasing customer expectations, loss due to process deviation, and process drift. Customer expectations are operationalized by tighter product specifications. The Taguchi loss function is employed to estimate the loss due to target deviation. In this paper we characterize the machine drift, uncertainty about future technological change, and the learn-and-break-in process by a generalized Brownian-Motion-Ito-Process. The photolithography process is analyzed in numerous scenarios varying demand, rejection levels, and quality losses.

1. Introduction

As customers demand higher quality and capability, greater precision and increased consistency are demanded of manufacturers. Manufacturers have passed that need down to equipment suppliers, who have expended considerable efforts to improve their products. From metal chipping to plastics to semiconductors, today's manufacturing equipment far surpasses older machines in both capability and cost. Therefore, the trade-off between machine capability and cost is a critical engineering and operations management decision.

A manager has two distinct equipment management options to cope with this capability/cost trade-off. The first involves replacing existing equipment with modern machines. The company incurs a significant purchase cost but weighs that against greater efficiencies and/or the probability of higher revenue from offering improved products to the market. The second option amounts to maintaining older equipment, usually resulting in tremendous savings in purchase costs over equipment replacement, but revenues may be lost from customers seeking better quality or consistency. A third option may become more commonplace as manufacturing equipment moves to a modular structure. In this case, only part of the machine is upgraded to achieve better capabilities. Functions such as controls or motors are replaced for a fraction of the cost of new equipment. This prolongs equipment life but may not achieve the same performance improvement as total replacement. However, it can accomplish dramatic process improvement in a timelier manner than waiting for equipment to be entirely redesigned.

This study explores the trade-off between equipment cost and capability under scenarios of increasing customer expectations. Expectations are operationalized by the part specification measure; higher customer expectations equate to tighter product specifications, which require higher capabilities of the process. Although saving on changeover costs, older equipment produces lower yields. In addition to the difficulty of managing higher user expectations, the concept of equipment aging is introduced. The production process is assumed to drift in a random manner over time as equipment wears. The drift is influenced by so many factors that it becomes unpredictable and almost indistinguishable from normal process noise. The Wiener process is used to model this drift.

Over some planning horizon, two categories of costs are incurred, equipment and quality. Equipment costs involve maintenance, replacement, and salvage value. Rejection costs for parts outside of specifications are commonly considered costs of poor quality, but even parts meeting specifications may yield costs or lost revenue as a critical dimension misses its target. Modeled as a Taguchi loss function (Taguchi *et al.*, 1989), this cost can be considerable when exacting manufacturing methods are required.

In an industry where user demands translate into consistently better performance and therefore tighter specifications, one would expect to see significant evaluation given to new equipment. As an example of such an industry consider computer hardware, where faster processing speed, larger memory in a smaller space, and greater bandwidth are resulting in the need for products, and therefore manufacturing equipment, with better capabilities. Semiconductor manufacturing equipment provides the basis for a quantitative analysis later in the paper.

The decision to replace or maintain equipment has been reported in the literature since Hotelling (1925) began looking at machine depreciation. A common approach to solving the problem, that of dynamic programming, dates back to Bellman (1955) and has since been followed by many others including Chand and Sethi (1982), Goldstein *et al.* (1988) and Gupta and Majumdar (1991). It is also common to assume that replacement equipment has a technological advantage over its predecessor. The difficulty here is making assumptions about how advanced future equipment will be. Sethi and Chand (1979) pursued the practice of determining a policy for an infinite planning horizon. Variations on this approach include the consideration of setup cost (Chand *et al.*, 1993), technological forecasts and technology improvement that are non-stationary in time (Nair and Hopp, 1992), determining error bounds on the finite horizon plan (Bean *et al.*, 1994), and adding the option of overhauling equipment versus routine maintenance (Karsak and Tolga, 1998).

By way of contrast, we assume technological change cannot keep pace with customer demand so predicting technological advance in equipment is moot. Technological change is patterned deterministically based upon the historical rate of change over the last 30 years. Planning horizons are determined by machine obsolescence, or in other words, technological change. The machine replacement policy takes into account the cost trade-off when considering the expense of new equipment, rejection and rework loss, and opportunity loss of imperfect quality. As distinct from past research, we have included two contemporary themes in this study: quality and rapid technological improvement. Quality is measured by yield and process capability. Markets have demanded rapid technological improvement in many products and equipment manufacturers are investing heavily to provide the tools to achieve the need. We model this by changing the production requirements on an ongoing basis over the planning horizon. In the next section we formulate a model for the machine replacement problem that takes into consideration quality loss from rejections. Finally, the implications of these results for equipment manufacturers are discussed.

2. Methodology

To facilitate the description of the model, we define the following notation.

$\{Y_n(t), t \geq 0\}$ = underlying production process (a stochastic process) of machine n with a known and non-random initial state $y_{n0} = Y_n(0)$;
$m(t)$ = time-variant target;
a_n = expected drift parameter of $Y_n(t)$;
b_n = uncertainty parameter that is the standard deviation of expected drift term of $Y_n(t)$;
$dZ(t)$ = standard Wiener process (Dixit and Pindyck, 1994);
$y_{ns} = Y_n(s)$ = initial value of underlying process $Y_n(t)$ at starting time s;
$\hat{y}_{ns} = E[y_{ns}]$ = mathematical expectation of y_{ns};
$LR(t, s, y_{ns})$ = rejection loss of the underlying process $Y_n(t)$ with initial value y_{ns};
$LQ(t, s, y_{ns})$ = quadratic deviation loss (or Taguchi loss) of underlying process $Y_n(t)$ with initial value y_{ns};
$C\{\mathrm{Var}Y_n(t), \mathrm{Var}Y(t)\}$ = variance reduction cost function.

As a starting point, let us simplify the problem of machine replacement to that of looking at the impacts on only one machine. The machine requires regular maintenance to maintain desired process results. Maintenance costs are assumed to increase each year over the life of the machine due to wear and tear. Over time, the machine's ability to hit the target on critical dimensions will deteriorate. Of course, continuous improvements efforts can ameliorate this machine aging, but will come at a cost. That cost will logically increase as the machine gets older. This framework is consistent with past research (Chand *et al.*, 1993).

On a periodic basis, this machine may be replaced with another more capable and more expensive machine. The replaced machine will have some salvage value at that time. After each time period, the specification limits on the production (machining) process will be tightened by a constant percentage to reflect higher user expectations. These customer demands are assumed to coincide with the introduction of new products or components of new products that are made from these machines.

As a result, firms are challenged with the managerial tasks of when and how to adjust the underlying production process to keep pace with fast-changing market needs. In this paper, such an interactive regulation process is characterized by a stochastic system containing a regulated drift, a time-variant target, and a state-dependent Wiener disturbance.

After adjustments, there will be a process for the underlying system to "learn-and-break-in" (Lieberman, 1989). The learn-and-break-in process will endure uncertainty in various situations such as performance fluctuation, quality deviation, and other exogenous disturbances such as temperature variation, raw material variation, input-voltage, and noise factors, which can cause non-uniformity in the production process. The result will be deviation from the target or desired value on critical dimensions. Therefore those uncertainty factors and the actual time needed for the break-in, which varies randomly and is dependent of the state, should be considered.

Assume the production of a new product model starts at time zero. Let $\{Y_n(t), t \geq 0\}$ with its state space on an interval $I \subseteq R$ denote the underlying regulated process, which is continuously measured and compared with the target $m(t)$. In an infinitesimal time interval after the adjustment is applied, the resulting improvement is expected to be $a_n(t)Y_n(t)\mathrm{d}t$. However, due to an uncertain disturbance, assumed to be Wiener type in the form of $b_n(t)Y(t)\mathrm{d}Z(t)$, the actual changes in the process $\mathrm{d}Y_n(t)$ contain an expected correction (drift) superimposed with an uncertain influence (disturbance), i.e.,

$$\mathrm{d}Y_n(t) = a_n(t)Y_n(t)\mathrm{d}t + b_n(t)Y_n(t)\mathrm{d}Z(t), \qquad (1)$$

where $\mathrm{d}Z(t)$ is a standard Wiener process, a_n and b_n are measurable and bounded on the interval $t \in (0, T]$.

In this paper we characterize the time-variant target of a process to be denoted by $m(t)$ and the target is assumed to be non-negative and bounded and consider S-type tolerances (Taguchi *et al.*, 1989), of which noise level, machine accuracy, shrinkage, wear, and deterioration are examples.

Proposition 1. *Let* $0 \leq s < t$, *and* $t \in [s, T]$ *where* T = *ending time, and* s = *starting time. Assume the initial status,* y_{n0}, *of the process is known and let* $\hat{y}_{ns} = E[y_{ns}]$, $\hat{y}^2_{ns} = E[y^2_{ns}]$. *Then the underlying process* (1) *can be expressed as follow:*

$$Y_n(t) = y_{ns} \exp\left[\left(a_n - \frac{b_n^2}{2}\right)(t-s) + b_n\{Z(t) - Z(s)\}\right],$$

$$E[Y_n(t)] = \hat{y}_{ns}\mathrm{e}^{a_n(t-s)} = y_{n0}\mathrm{e}^{a_n t},$$

$$E[Y_n^2(t)] = \hat{y}^2_{ns}\mathrm{e}^{(2a_n + b_n^2)(t-s)} = y^2_{n0}\mathrm{e}^{2a_n t + b_n^2 t},$$

$$\mathrm{Var}[Y_n(t)] = \mathrm{e}^{2a_n(t-s)}\left\{\hat{y}^2_{ns}\mathrm{e}^{b_n^2(t-s)} - (\hat{y}_{ns})^2\right\}$$
$$= (y_{n0})^2\mathrm{e}^{2a_n t}\{\mathrm{e}^{b_n^2 t} - 1\},$$

where $Z(t)$ *is a Wiener process (or Brownian motion).*

Proof. By Theorem 8.4.2 in Arnold (1992), the integral (in the Ito sense) as defined in (1) with starting time $t = s$ is given by

$$Y_n = y_{ns} \exp\left[\left(a_n - \frac{b_n^2}{2}\right)(t-s) + b_n\{Z(t) - Z(s)\}\right].$$

Since $\hat{y}_{ns} = y_{n0}\mathrm{e}^{a_n s}$ and $\hat{y}^2_{ns} = y^2_{n0}\mathrm{e}^{(2a_n + b_n^2)s}$ by Theorem 8.4.5 in (Arnold, 1992), then $E\{Y_n(t)\} = \hat{y}_{ns}\mathrm{e}^{a_n(t-s)} = y_{n0}\,\mathrm{e}^{a_n t}$ and $E\{Y_n^2(t)\} = \hat{y}^2_{ns}\mathrm{e}^{[(2a_n + b_n^2)(t-s)]} = y^2_{n0}\mathrm{e}^{(2a_n + b_n^2)t}$ ■

2.1. *Rejection loss*

Now consider the cost of quality impact on this analysis. For any part dimension, a machine's output will vary on a measure of that dimension from item to item. This is referred to as chance, or random, variation. Occasionally, that variation may exceed some prescribed value, called a specification limit, at which point the output is deemed inappropriate for use. It may be scrapped or reworked to bring it back within specifications, but at a cost to the company. This cost will be referred to as rejection loss. It is assumed that the manufacturing process and specification limits are symmetrical around the desired target and rejection loss per part is constant regardless of the direction or magnitude of the defect. We define specification limits that are tightening over time as follows:

$l(t)$ = the lower specification limit associated with time t;
$u(t)$ = the upper specification limit associated with time t.

Theorem 1. *Let* $y_{ns} = Y_n(s)$, *where* $s < t \leq T$, *then*

$$\Pr[l(t) \leq Y_n(t) \leq u(t) | Y_n(s) = y_{ns}]$$
$$= \left[\frac{1}{\sqrt{2\pi b_n^2(t-s)}}\left\{\int_{Z_l}^{Z_u} \exp\left(\frac{-y^2}{2(t-s)b_n^2}\right)\mathrm{d}y\right\}\right],$$

where

$$z_u = \ln\left\{\frac{u(t)}{y_{ns}}\right\} - \left(a_n - \frac{b_n^2}{2}\right)(t-s),$$

$$z_l = \ln\left\{\frac{l(t)}{y_{ns}}\right\} - \left(a_n - \frac{b_n^2}{2}\right)(t-s), \text{ and } s < t \leq T.$$

Therefore the rejection loss of the underlying process $Y_n(t)$ *with initial value* y_{ns} *based on specification limits* $l(t)$ *and* $u(t)$, *denoted by* $LR(t, s, y_{ns})$, *can be expressed as follows:*

$$LR(t, s, y_{ns}) = C\left[1 - \frac{1}{\sqrt{2\pi b_n^2(t-s)}}\left\{\int_{Z_l}^{Z_u} \exp\left(\frac{-y^2}{2(t-s)b_n^2}\right)\mathrm{d}y\right\}\right]$$

where C *is the cost of defective production over the time interval between periods.*

Proof. Since

$$\Pr[l(t) \leq Y_n(t) \leq u(t) | Y_n(s) = y_{ns}]$$
$$= \left[\frac{1}{\sqrt{2\pi b_n^2(t-s)}}\left\{\int_{Z_l}^{Z_u} \exp\left(\frac{-y^2}{2(t-s)b_n^2}\right)\mathrm{d}y\right\}\right],$$

where

$$z_u = \ln\left\{\frac{u(t)}{y_{ns}}\right\} - \left(a_n - \frac{b_n^2}{2}\right)(t-s),$$

and

$$z_l = \ln\left\{\frac{l(t)}{y_{ns}}\right\} - \left(a_n - \frac{b_n^2}{2}\right)(t-s),$$

by example 9.2.13 in Arnold (1992),

$$LR(t,s,y_{ns}) = C[1 - \Pr\{l(t) \le Y_n(t) \le u(t)|y_{ns}\}]$$
$$= C\left[1 - \frac{1}{\sqrt{2\pi b_n^2(t-s)}}\left\{\int_{z_l}^{z_u} \exp\left(\frac{-y^2}{2(t-s)b_n^2}\right)\mathrm{d}y\right\}\right],$$

and $s < t \le T$. ■

Suppose we only know the initial value of each new machine at starting time $Y_n(0) = y_{n0}$. We can estimate $\hat{y}_{ns} = E[Y_n(s)]$, of an expected initial value of an old machine that starts at $0 < s \le T$. The following corollary can be used for the estimate of $LR(t,s,\hat{y}_{ns})$ with underlying process $Y_n(t)$ starting at s.

Corollary 1. *Let the initial value y_{n0} be known at starting time $t = 0$, then*

$$\Pr[l(t) \le Y_n(t) \le u(t)|Y_n(s) = \hat{y}_{ns}]$$
$$= \left[\frac{1}{\sqrt{2\pi b_n^2(t-s)}}\left\{\int_{\hat{z}_l}^{\hat{z}_u} \exp\left(\frac{-y^2}{2(t-s)b_n^2}\right)\mathrm{d}y\right\}\right], \quad (2)$$

where

$$\hat{z}_u = \ln\left\{\frac{u(t)}{\hat{y}_{ns}}\right\} - \left(a_n - \frac{b_n^2}{2}\right)(t-s)$$
$$= \ln\left\{\frac{u(t)}{y_{n0}}\right\} - a_n t - \frac{b_n^2}{2}(t-s),$$
$$\hat{z}_l = \ln\left\{\frac{l(t)}{\hat{y}_{ns}}\right\} - \left(a_n - \frac{b_n^2}{2}\right)(t-s)$$
$$= \ln\left\{\frac{l(t)}{y_{n0}}\right\} - a_n t + \frac{b_n^2}{2}(t-s).$$

and $0 < s \le t \le T$.

Therefore the rejection loss $LR(t,s,\hat{y}_{ns})$ based on specification limits $l(t)$ and $u(t)$ can be expressed as:

$$LR(t,s,\hat{y}_{ns}) = C\left[1 - \frac{1}{\sqrt{2\pi b_n^2(t-s)}}\left\{\int_{\hat{z}_l}^{\hat{z}_u} \exp\left(\frac{-y^2}{2(t-s)b_n^2}\right)\mathrm{d}y\right\}\right].$$

Proof. Since

$$\hat{y}_{ns} = y_{n0}e^{a_n s},$$
$$\hat{z}_u = \ln\left\{\frac{u(t)}{y_{n0}}\right\} - a_n t + \frac{b_n^2}{2}(t-s),$$

and

$$\hat{z}_l = \ln\left\{\frac{l(t)}{y_{n0}}\right\} - a_n t + \frac{b_n^2}{2}(t-s),$$

by Theorem 1. ■

2.2. *Quadratic deviation loss from target (Taguchi loss)*

As suggested by Taguchi, *et al.* (1989), a quadratic loss function can be constructed to reflect the loss of deviation from the target, $m(t)$, as following,

$$l\{Y(t)\} = A\{Y(t) - m(t)\}^2.$$

Here, A is the cost per unit of deviation and $m(t)$ is the target that is assumed to be *a priori* with the following characteristics: positive and bounded, time-variant, and piecewise uniformly continuous.

Theorem 2. *The expected quadratic loss of the underlying process, $Y_n(t)$, is denoted by*

$$LQ(t,s,\hat{y}_{ns}) = A\left[y_{n0}^2 e^{(2a_n+b_n^2)t} - 2m(t)y_{n0}e^{a_n t} + m^2(t)\right]. \quad (3)$$

Proof. Since

$$LQ(t,s,\hat{y}_{ns}) = E[l\{Y_n(t)\}]$$
$$= A\left[E\{Y_n^2(t)\} - 2m(t)E\{Y_n(t)\} + m^2(t)\right],$$

$$LQ(t,s,\hat{y}_{ns}) = A\left[\hat{y}_{ns}^2 e^{(2a_n+b_n^2)(t-s)} - 2m(t)\hat{y}_{ns}e^{a_n(t-s)} + m^2(t)\right],$$
$$= A\left[y_{n0}^2 e^{(2a_n+b_n^2)t} - 2m(t)y_{n0}e^{a_n t} + m^2(t)\right]$$

by Proposition 1.

2.3. *Variance reduction*

In response to increasing expectations from users, management has the ability to improve the machine's capability, at a cost of course. This falls under the guise of continuous improvement but its cost here will be assumed to occur on a periodic basis, to coincide with the maintain/replace decision. Therefore, if management decides to maintain rather than replace the machine, a reduction in the process variance will reduce the expected rejection loss and Taguchi loss under the tighter specification limits. We assume the variance reduction cost will increase quadratically to reflect the difficulty in achieving an ever-higher process capability. We use the following functional form for the variance reduction cost function:

$$C\{\mathrm{Var}(Y_n), \mathrm{Var}(Y)\} = \frac{P}{\mathrm{Var}(Y_n)}\{\mathrm{Var}(Y_n) - \mathrm{Var}(Y)\}^2, \quad (4)$$

where

$$\mathrm{Var}[Y_n(t)] = e^{2a_n(t-s)}\left\{\hat{y}_{ns}^2 e^{b_n^2(t-s)} - (\hat{y}_{ns})^2\right\},$$
$$= (y_{n0})^2 e^{2a_n t}\left\{e^{b_n^2(t)} - 1\right\}(\text{Current variance}),$$

$$\text{Var}[Y(t)] = e^{2a(t-s)}\left\{\hat{y}_{ns}^2 e^{b_n^2(t-s)} - (\hat{y}_{ns})^2\right\} \text{ (Target Variance)},$$

where P is a constant to reflect the cost of process improvement. Given the specification limits $l(t)$ and $u(t)$ and the current variance $\text{Var}(Y_n)$, in order to maximize the net gain from a variance reduction exercise means:

Net gain in the variance reduction operation = Quadratic deviation and rejection loss before variance reduction − Quadratic deviation and rejection loss after reduction − Variance reduction cost,

$$\begin{aligned} &= LR\{\text{Var}(Y_n), u(t), l(t)\} + LQ\{\text{Var}(Y_n), u(t), l(t)\} \\ &\quad - LR\{\text{Var}(Y), u(t), l(t)\} - LQ\{\text{Var}(Y), u(t), l(t)\} \\ &\quad - C\{\text{Var}(Y_n), \text{Var}(Y)\}. \end{aligned}$$

Maximizing this gain is equivalent to minimizing the following expression

$$\begin{aligned} &\underset{a}{\text{Min}}[LR\{\text{Var}(Y), u(t), l(t)\} + LQ\{\text{Var}(Y), u(t), l(t)\} \\ &\quad + C\{\text{Var}(Y_n), \text{Var}(Y)\}. \end{aligned}$$

Let $S(a, b_n) = LR\{\text{Var}(Y), u(t), l(t)\} + LQ\{\text{Var}(Y), u(t), l(t)\} + C\{\text{Var}(Y_n), \text{Var}(Y)\}$ and define a^* as a value of a satisfying the following equations:

$$S_a(a, b_n) = 0 \text{ and } S_{aa}(a, b_n) > 0.$$

Theorem 3. *If a^* exists, then the optimal variance, $Var(Y) = e^{2a^*(t-s)}\left\{\hat{y}_{ns}^2 e^{b_n^2(t-s)} - (y_{ns})^2\right\}$, which minimizes the total cost $S(a, b_n)$ with $a = a^*$ for all $t \in (s, T]$,*

where

$$S_a(a, b_n) = LR_a(a, b_n) + LQ_a(a, b_n) + C_a(a, b_n), \quad \textit{and}$$
$$S_{aa}(a, b_n) = LR_{aa}(a, b_n) + LQ_{aa}(a, b_n) + C_{aa}(a, b_n).$$

$$LR_a = C\left(\sqrt{\frac{t-s}{2\pi b_n^2}}\right)\left[\exp\left(\frac{-Z_u^2}{2b_n^2(t-s)}\right) - \exp\left(\frac{-Z_l^2}{2b_n^2(t-s)}\right)\right],$$

$$LR_{aa} = \left(\sqrt{\frac{t-s}{2\pi b_n^2}}\right)\left(\frac{C}{b_n^2}\right) \times \left\{Z_u \times \exp\left(\frac{-Z_u^2}{2b_n^2(t-s)}\right) - Z_l \times \exp\left(\frac{-Z_l^2}{2b_n^2(t-s)}\right)\right\},$$

$$LQ_a = 2A(t-s)\left[\hat{y}_{ns}^2 e^{(2a+b_n^2)(t-s)} - m(t)\hat{y}_{ns}e^{a(t-s)}\right],$$

$$LQ_{aa} = 2A(t-s)^2 e^{a(t-s)}\left[2\hat{y}_{ns}^2 e^{(a+b_n^2)(t-s)} - m(t)\hat{y}_{ns}\right],$$

$$C_a = -4P(t-s)\text{Var}(Y)\left[1 - \frac{\text{Var}(Y)}{\text{Var}(Y_n)}\right],$$

$$C_{aa} = 8P(t-s)^2\text{Var}(Y)\left\{\frac{2\text{Var}(Y)}{\text{Var}(Y_n)} - 1\right\}.$$

Proof. Since

$$\begin{aligned} S(a, b_n) &= LR\{\text{Var}(Y), u(t), l(t)\} + LQ\{\text{Var}(Y), u(t), l(t)\} \\ &\quad + C\{\text{Var}(Y_n), \text{Var}(Y)\}, \end{aligned}$$

$$S_a = \frac{\partial S}{\partial a} = LR_a + LQ_a + C_a,$$

and

$$S_{aa} = \frac{\partial^2 S}{\partial a^2} = LR_{aa} + LQ_{aa} + C_{aa}.$$

By Leibniz's rule,

$$LR_a = C\left(\sqrt{\frac{t-s}{2\pi b_n^2}}\right)\left[\exp\left(\frac{-Z_u^2}{2b_n^2(t-s)}\right) - \exp\left(\frac{-Z_l^2}{2b_n^2(t-s)}\right)\right].$$

Hence

$$LR_{aa} = \left(\sqrt{\frac{t-s}{2\pi b_n^2}}\right)\left(\frac{C}{b_n^2}\right) \times \left\{Z_u \times \exp\left(\frac{-Z_u^2}{2b_n^2(t-s)}\right) - Z_l \times \exp\left(\frac{-Z_l^2}{2b_n^2(t-s)}\right)\right\}.$$

And $LQ_a = 2A(t-s)\left[\hat{y}_{ns}^2 e^{(2a+b_n^2)(t-s)} - m(t)\hat{y}_{ns}e^{a(t-s)}\right]$.

Hence $LQ_{aa} = 2A(t-s)^2 e^{a(t-s)}\left[2\hat{y}_{ns}^2 e^{(a+b_n^2)(t-s)} - m(t)\hat{y}_{ns}\right]$.

Since

$$\frac{\partial \text{Var}(Y)}{\partial a} = 2(t-s)\text{Var}(Y),$$

$$C_a = -4P(t-s)\text{Var}(Y)\left[1 - \frac{\text{Var}(Y)}{\text{Var}(Y_n)}\right].$$

Hence

$$C_{aa} = 8P(t-s)^2\text{Var}(Y)\left\{\frac{2\text{Var}(Y)}{\text{Var}(Y_n)} - 1\right\}.$$

Therefore if there exist a^* that satisfies $S_a(a^*, b_n) = 0$ and $S_{aa}(a^*, b_n) > 0$, then $S(a, b_n)$ has minimum total cost at $a = a^*$. ∎

Corollary 2. *Under the-smaller-the-better (S-type) tolerance, the underlying process has initial value $y_{n0} > 0$ with $a_{n+1} \le a \le a_n$, the target value $m(t) = 0$ for all $t \in (0, T]$ and the upper tolerance limit is $u(t)$. If $A \times$ (Number of units) $\ge 2P$, then $S(a, b_n)$ has a minimum cost at $a^* = a_{n+1}$.*

Proof. Since $m(t) = 0$,

$$LR_a = C\left(\sqrt{\frac{t-s}{2\pi b_n^2}}\right)\left[\exp\left(\frac{-Z_u^2}{2b_n^2(t-s)}\right)\right] > 0,$$

and

$$LQ_a = 2A(t-s)\left[\hat{y}_{ns}^2 e^{(2a+b_n^2)(t-s)}\right] > 0.$$

Since

$$\left[1 - \frac{\text{Var}(Y)}{\text{Var}(Y_n)}\right] < 1,$$

$$\text{Min } C_a \geq -4P(t-s)\text{Var}(Y) \geq -4P(t-s)\left\{e^{2a(t-s)}\right\} \times \left\{\hat{y}_{ns}^2 e^{b_n^2(t-s)}\right\}.$$

Therefore $LQ_a + \{\text{Min}C_a\} \geq 2(t-s)\{A \times (\textit{Number of units}) - 2P\}\left\{\hat{y}_{ns}^2 e^{(-2a+b_n^2)(t-s)}\right\} > 0$. Hence if $\{A \times (\textit{Number of units}) - 2P\} > 0$, then $LQ_a + C_a > 0$. Therefore $S(a,b_n) > 0$. This implies that $S(a,b_n)$ is a strictly increasing function of a. Hence $S(a,b_n)$ has a minimum cost at $a^* = a_{n+1}$ if $A \times (\textit{Number of units}) > 2P$. ■

3. Example

Take the computer hardware industry as an example of where demand for greater performance is readily evident. Whether it is telecommunications, 3-D modeling, data mining, gaming, or any other area of computing, performance depends on the capabilities of integrated circuits. To illustrate the advancement in integrated circuit capability, in 1978 Intel's 8086 microprocessor packed about 20 000 transistors onto a chip. Today, Intel's Pentium IV microprocessor unit has nearly 42 million transistors (Anon, 2002). That progression of density is expected to grow at a compounded rate of 50% per year (Anon, 1997). If specifications are tightened at that pace, the future need for machines with improved capabilities is staggering.

The semiconductor manufacturing industry spent approximately \$20 billion on equipment alone in 1997 (Dorsch, 1998) and was forecasted to spend nearly \$26 billion in 2000 (Anon, 1999). The size of the semiconductor equipment industry now exceeds the size of the cutting, rolling, and paper making machinery industries. Semiconductor manufacturers like Intel adopt new generations of leading equipment as soon as equipment manufacturers like Applied Materials deliver them. The burden on equipment manufacturers to keep pace with customer demands has resulted in significant research investment. One area, photolithography, exemplifies this. Photolithography is the use of light to transfer a pattern or image from one medium to another, as from a mask to a wafer. The equipment to perform this function has undergone numerous breakthroughs in process technology just over the last 10 years and represents 30–40% of total fabrication plant costs (Seligson, 1998). Cutting edge equipment has about a 5-year useful life before it is relegated to non-critical layers or commodity chips.

Yet operating the equipment involves constant monitoring and experimentation to control and improve output levels. As mentioned earlier, this research chooses a Wiener process to model drift attributable to a variety of controllable and uncontrollable factors. In photolithography, production factors include temperature, humidity, resist sensitivity, contrast, process latitude, field size, optical source, dose, mask age, and many more. If slight changes in one variable can have unpredictable effects on the process, managing the many changes of a production environment may lead to random or drifting results.

The following example specifically addresses the photolithography market under S-type control. First, we assume a critical dimension (like IP placement) has target value $m(t) = 0$, upper specification limit of 7.5, 7% of output rejected, and initial value of the process $y_{n0} = 0.05$. Each year user demands increase, which translates into a 10% reduction in the upper specification limit. This reduction rate is conservative compared to historical progression (Anon, 1997). Fortunately, each year one of the photolithography equipment manufacturers introduces a new machine that exactly matches this percentage change. In addition, this manufacturer has decreased the b_n parameter by 10%. In essence, machinery advances keep pace exactly with user demands. Over a 5-year span, the data are presented in Table 1.

The output of photolithography is an etched silicon wafer. The value of a defective wafer depends upon many factors, such as the number of chips, application of chips, and degree of completion, but most wafers made defective in photolithography can be reworked. Rework involves stripping off a coating of resist, reapplying the resist, and then sending the wafer back through the etching process. Using a 25 × 25 mm die size (chip size), approximately 100 chips can be etched on a 300 mm wafer. If those chips are microprocessors or ASICs (Application Specific Integrated Circuits), a cost of \$150/chip is not out of the question (Muzio *et al.*, 1999). Assuming

Table 1. Model parameters

	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Upper specification limit $\mathbf{u(t)}$	7.5	6.75	6.075	5.4675	4.920 75
a_n	4.397 73	4.342 42	4.284 23	4.223 06	4.158 87
b_n	0.5	0.45	0.405	0.3645	0.328 05
y_{n0}	0.05	0.05	0.05	0.05	0.05
$\sigma_n^2 = \text{Var}(Y_n)$	4.689	3.317 85	2.345 25	1.654 28	1.163 45
Rejection percentage, (%)	7	7	7	7	7

production of microprocessors, we conservatively assume a cost of \$40 per chip. Even moderately performing microprocessors sell for at least twice that when purchased in large quantities. Therefore, a reworked wafer (rejection) would cost the company an extra processing charge plus about a 10% charge for stripping and reapplying the resist, or roughly \$4400. The fact that photolithography is a bottleneck in most fabs (Muzio *et al.* 1999) prevents much improvement over these numbers. As such, these machines run virtually day and night and although their annual output varies greatly, we use a value of 200 000 wafers per year suggested by the work of McGraw (1998).

Each year in our example, a company must decide to maintain or replace its equipment. New equipment prices for the beginning of year n are given in Table 2 and based on data from Anon (1998) and Gomei and Suzuki, (1998). It must be noted that actual equipment costs are highly guarded information in this industry, will greatly depend on volume discounts, and do not necessarily increase linearly. Salvage values for equipment at the beginning of year n are based on a Modified Accelerated Cost Recovery System (MACRS) with a 5-year straight-line depreciation (general depreciation system (McGraw, 1998)). Photolithography equipment requires regular maintenance during the week and that value is set at 7% of the equipment cost in the first year, and increases linearly (Anon, 1999; Muzio *et al.*, 1999).

Should a company decide to maintain equipment instead of replacing it, they may also perform maintenance (continuous improvement) to lower the machine's process variance and prevent greatly increased rejection losses in the subsequent year. Quadratic deviation (Taguchi) losses are related to the rejection cost and the specification limit so the constant A in Equation (3) is set to \$39 per unit in the first year. Microprocessor production provides a perfect example of losses related to missing the target. Slight deviations from critical dimensions can lower performance of chips significantly. The result could be a microprocessor with clock speed of 1.0 GigaHertz as opposed to a desired 1.4 GigaHertz. The revenue lost from such a "miss" can be in the hundreds of dollars per chip. Therefore, process capability relates directly to revenue via performance achievements.

The process improvement cost, P in Equation (4), is set to \$1000 000 based on data obtained from Heerssen (1999) and McIntosh (1999) and new equipment prices. Using Theorem 3, optimal variances were calculated for new and maintained equipment during years 2–5 of the planning horizon and are included in Table 3. Recall that for new equipment the optimal variance is already achieved, although some rejected material will still be realized. The expected cost of achieving that optimal variance is given in Table 4, expected rejection losses based on process variance in Table 5, and loss from target deviation in Table 6. Finally, Table 7 shows a summary of quality-related costs.

Using dynamic programming and the equipment-related costs mentioned earlier, the optimal equipment policy under these circumstances involves replacing equipment on an annual basis. Quality-related costs (rejections and missing the target) dwarf the expense of new equipment. Understanding that practitioners may not quantify some of the modeled concepts, we extend the

Table 2. New equipment prices (in \$ millions)

Age	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
New	5	5.65	6.3845	7.2145	8.1524

Table 3. Variances for new and maintained equipment

Age	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
New	4.6894	3.317 85	2.345 25	1.654 28	1.163 45
1		4.198 32	2.953 34	2.075 19	1.454 97
2			3.737 08	2.613 26	1.825 17
3				3.306 74	2.298 41
4					2.908 34

Table 4. Optimal variance reduction cost (in \$ millions)

Age	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
New	0	0	0	0	0
1		0.051 43	0.040 05	0.0311	0.024 01
2			0.050 67	0.039 16	0.030 12
3				0.049 56	0.037 93
4					0.048

Table 5. Rejection loss with optimal variance reduction (in \$ millions)

Age	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
New	61.6	61.6000	61.6000	61.6000	61.6000
1		74.3248	74.9672	75.5568	76.0936
2			88.0264	89.3640	90.5784
3				102.4672	104.5000
4					117.3568

Table 6. Quadratic deviation (Taguchi) loss (in \$ millions)

Age	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
New	135.0302	143.9864	154.0290	164.9126	176.4126
1	0	146.1268	154.5952	164.1296	174.4630
2	0	0	157.4096	165.1458	173.9478
3	0	0	0	168.8016	175.5864
4	0	0	0	0	180.2414

Table 7. Total quality-related costs (in $ millions)

Age	n = 1	n = 2	n = 3	n = 4	n = 5
New	201.9802	211.6319	222.4604	234.2321	246.7356
1	0	220.8775	230.0256	240.1957	251.1210
2	0	0	245.8874	255.0018	265.0680
3	0	0	0	271.7471	280.6088
4	0	0	0	0	298.1050

Table 8. Total quality-related costs (in $ millions)

Age	n = 1	n = 2	n = 3	n = 4	n = 5
New	28.45	29.1455	29.931 42	30.8195	31.823 03
1		28.297 73	28.575 94	28.8431	29.099 48
2			33.461 29	34.003 47	34.508 69
3				38.903 53	39.709 94
4					44.515 58

analysis to the production of DRAMs, somewhat of a commodity chip. Production volumes are higher, chips have less value, and close adherence to critical dimensions does not have a great influence on performance. Therefore, we eliminate quality losses attributable to being within specifications but away from the target, as characterized by the Taguchi loss function, and modify parameters as follows: unit rejection cost, $C = \$550$ and production = 600 000 wafers per year. Equipment-related costs remain the same from the first example. This scenario results in Table 8.

Here, again, the optimal equipment policy would be to replace equipment at the beginning of each year. Knowing that semiconductor manufacturers do not replace equipment this frequently, we conducted a series of experiments varying demand, rejection cost (C), Taguchi cost, and yield (rejection percentage). The resulting machine replacement policies are given in Tables 9, 10, and 11 (M means maintain and R means replace).

4. Implications for equipment manufacturers

As is evident from the tables, many scenarios do exist where annual machine replacement is not optimal. Changing the rejection percentage from 7 to 4 to 2% had very little impact on replacement policies. That percentage can vary from batch to batch on a photolithography machine so having some robustness to the solution provides some external validity. Eliminating Taguchi loss resulted in no policy change in 16 of 18 comparisons although the difference in total costs was huge. Dropping Taguchi loss from the analysis makes our model simpler to apply and apparently without much loss of optimality for commodity chip production. Further analysis would be required for more expensive semiconductors. When reducing demand by one-third and two-thirds, far fewer replacements are required. Of course, maintenance is the fiscally responsible action during leaner times. Lowering rejection cost also produced drastic policy differences. At the lowest volume and lowest rejection cost, maintenance was the best decision in all 24 opportunities. For commodity chips costing less than a dollar each, and no

Table 9. Machine replacement policies varying demand and quality costs with 7% rejection

Demand	C	Taguchi loss	Total cost	Policy after years 1–4
600 000	550	no	130.77	RRRR
		yes	421.16	RRRR
	300	no	78.27	RRRR
		yes	236.65	RRRR
	100	no	34.69	RRMM
		yes	87.26	RRMM
400 000	550	no	92.27	RRRR
		yes	285.26	RRRR
	300	no	56.86	RRRM
		yes	162.18	RRRM
	100	no	26.69	RMMM
		yes	61.76	RRMM
200 000	550	no	53.20	RRRM
		yes	149.75	RRRM
	300	no	34.69	RRMM
		yes	87.26	RRMM
	100	no	17.30	MMMM
		yes	35.19	MMMM

Table 10. Machine replacement policies varying demand and quality costs with 4% rejection

Demand	C	Taguchi loss	Total cost	Policy after years 1–4
600 000	550	no	81.27	RRRR
		yes	327.18	RRRR
	300	no	51.27	RRRR
		yes	185.18	RRRR
	100	no	25.28	RRMM
		yes	69.76	RRMM
400 000	550	no	59.27	RRRR
		yes	231.21	RRRR
	300	no	38.56	RRRM
		yes	127.75	RRRM
	100	no	20.21	RMMM
		yes	49.91	RMMM
200 000	550	no	36.42	RRRM
		yes	118.18	RRRM
	300	no	25.28	RRMM
		yes	69.76	RRMM
	100	no	13.91	MMMM
		yes	28.94	MMMM

Table 11. Machine replacement policies varying demand and quality costs with 2% rejection

Demand	*C*	*Taguchi loss*	*Total cost*	*Policy after years 1–4*
600 000	550	no	48.27	RRRR
		yes	250.27	RRRR
	300	no	32.67	RRRM
		yes	142.69	RRRM
	100	no	18.55	RRMM
		yes	55.19	RRMM
400 000	550	no	37.07	RRRM
		yes	171.54	RRRM
	300	no	26.07	RRRM
		yes	99.42	RRRM
	100	no	15.35	RMMM
		yes	39.82	RMMM
200 000	550	no	24.97	RRRM
		yes	92.20	RRRM
	300	no	18.55	RRMM
		yes	55.19	RRMM
	100	no	11.26	MMMM
		yes	23.61	MMMM

Taguchi loss assumed, not only was annual maintenance optimal but also the savings over alternative 4-year policies (as measured by total cost) was large.

Further analysis of the cost data from these experiments yields an additional insight. The differences between an economically optimal policy and the worst alternative are in the range of 8 to 30% for Table 9 scenarios (8 to 11% without Taguchi loss). So even if a company chose to maintain equipment in an environment that suggested otherwise, the financial loss may be too small for companies to notice. Not modeled and possibly significant is the impact of equipment changeover on the production process, but such a cost could be reduced with learning. Certainly manufacturers have learned to reduce setup costs on lot changeovers, justifying smaller batch sizes, and the same may be said for equipment changeover. Also not modeled is cyclical demand, readily apparent in today's semiconductor market. We leave this extension to future research.

Still, given the many situations where frequent replacement is economically justified, there exists a strong need for better equipment capabilities. This may place tremendous pressure on equipment providers to upgrade equipment quickly or face significant loss of market share in just a short period of time. Given the cost and time to develop these new products, an efficient alternative would be to design the equipment in modules (Sanchez, 1996). Opening the module design up to a wide variety of sources encourages new approaches that often result in radical performance breakthroughs (Morris and Ferguson, 1993), exactly what may be needed in the environments modeled above. Philips Electronics COO Stuart McIntosh has suggested exactly this approach (McIntosh, 1998), stating, "Modular and upgradeable equipment will become necessary to high capital productivity in fabs."

5. Conclusion

In this paper, we have modeled the traditional equipment maintain or replace decision under scenarios of increasing customer expectations and economic awareness of quality. Through numerous examples, the model explores the trade-off of costs arising from rejection/rework and variation from critical machining dimensions versus the toll associated with purchasing new equipment or maintaining existing equipment. Although the results of our analysis, i.e., increasing expectations require better processes, may seem intuitive, the model provides practitioners with a method to assess a variety of factors (equipment aging, target deviation, changing demands) in the equipment replacement decision. For the most part, the data used in the photolithography manufacturing analysis are realistic and verifiable and indicate the need to replace equipment frequently. The opportunities and efficiencies for alternatives like modular equipment design and radical performance breakthroughs should eventually lead to their acceptance in the semiconductor equipment industry.

References

Anon (1997) *Status*, Integrated Circuits Engineering Corporation, Scottsdale, AZ.

Anon (1998) VLSI Research Corp., San Jose, CA.

Anon (1999) Semiconductor equipment monitor. *Semiconductor International*, **22**(7), 31.

Anon (2002) *Intel Microprocessor Quick Reference Guide*, at *http://www.intel.com/pressroom/kits/processors/quickreffam.htm#P4.*

Arnold, L. (1992) *Stochastic Differential Equations: Theory and Applications*, Wiley, New York, NY.

Bean, J., Lohmann, J. and Smith, R. (1994) Equipment replacement under technological change. *Naval Research Logistics*, **41**(1), 117.

Bellman, R.E. (1955) Equipment replacement policy. *The Society for Industrial and Applied Mathematics*, 3(3), 133–136.

Chand, S., McClurg, T. and Ward, J. (1993) Single-machine replacement model with learning. *Naval Research Logistics*, **40**(2), 175–192.

Chand, S. and Sethi, S. (1982) Planning horizon procedures for machine replacement models with several possible replacement alternatives. *Naval Research Logistics Quarterly*, **29**(3), 483–493.

Dixit, A.K. and Pindyck, R.S. (1994) *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ.

Dorsch, J. (1998) Retooling in '98. *Electronic Business*, **24**(1), 72–73.

Goldstein, T., Ladany, S.P. and Mehrez, A. (1988) A discounted machine-replacement model with an expected future technological breakthrough. *Naval Research Logistics*, **35**, 209–220.

Gomei, Y. and Suzuki, M. (1998) A cost-of-ownership study on lithography systems. *Semiconductor International*, **21**(8), 143–147.

Gupta, A.D. and Majumdar, P.K. (1991) Overhaul and replacement decisions for capital equipment: a case. *International Journal of Quality and Reliability Management*, **6**(4), 8–18.

Heerssen, G. (1999) Advanced process controls in high-volume IC manufacturing. *Future Fab*, **5**.

Hotelling, H. (1925) A general mathematical theory of depreciation. *Journal of the American Statistical Association*, **20**, 340–353.

Karsak, E. and Tolga, E. (1998) Overhaul-replacement model for equipment subject to technological change in an inflation-prone economy. *International Journal of Production Economics*, **56**(7), 291–301.

Lieberman, M.B. (1989) Learning productivity, and US-Japan industrial competitiveness. *Managing Internal Operations*, Ferdows, K. (ed.), North-Holland, Amsterdam.

McGraw, M. (1998) Lessons from fab woebegone. *Semiconductor International*, **21**(12).

McIntosh, S. (1998) Industry watch. *Semiconductor International*, **21**(7), 29.

McIntosh, S. (1999) Is the two billion dollar fab economic? *Future Fab*, **5**.

Morris, C.R. and Ferguson, C.H. (1993) How architecture wins technology wars. *Harvard Business Review*, **17**(2), 86–97.

Muzio, E., Seidel, P., Mason, M., Canning, J. and Shelden, G. (1999) *Lithography Cost of Ownership Analysis*, Revision Number 4.0, International SEMATECH Lithography Division.

Nair, S. and Hopp, W. (1992) Model for equipment replacement due to technological obsolescence. *European Journal of Operational Research*, **63**(2), 207–221.

Sanchez, R. (1996) Strategic product creation: managing new interactions of technology, markets, and organizations. *European Management Journal*, **14**(2), 121–138.

Seligson, D. (1998) The economics of 300 mm processing. *Semiconductor International*, **21**(1), 52–56.

Sethi, S. and Chand, S. (1979) Planning horizon procedures for machine replacement models. *Management Science*, **24**(2), 140–151.

Taguchi, G., Elsayed, E.A. and Hsiang, T.C. (1989) *Quality Engineering in Production Systems*, McGraw-Hill, New York, NY.

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Originally accepted as part of the Scheduling and Logistics Special Issue on Semiconductor Manufacturing