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Stochastic Business Cycle Volatilities, Capital Accumulation and Economic Growth: Lessons from the Global Credit Market Crisis

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Abstract

The recent global economic downturn in a number of economies was preceded by rising credit market risk brought on by a massive financial market failure. This paper develops a small open economy model that analyzes the interaction of business cycle volatilities with capital accumulation and the subsequent impacts on economic growth. We use a stochastic dynamic programming model to test the central hypothesis that rising volatility shocks is an inhibitor to capital accumulation and subsequently economic growth. The model illustrates that traditional capital-based growth models which assume a constant capital stock are not consistent with the business cycle variation in capital accumulation.

Furthermore, it appears that an increase in precautionary savings arising from a stochastic shock does not completely translate into productive capital investment need for growth, since risk-averse households will seek out risk-free government or foreign assets. We find this conclusion consistent with the empirical findings of Ramey et al (1995) and Badinger (2009) who both argued that, business cycle volatility is important to the growth discussion because of its robust net negative effect on output growth.

Stochastic Business Cycle Volatilities, Capital Accumulation and Economic Growth
: Lessons from the Global Credit Market Crisis

1.0   Introduction

The extant literature on economic growth can be broadly divided into two main endogenous sources of growth (see Romer (1986)). On the one hand, there are the capital-based models that are grounded in the notion that growth is through the endogenous accumulation of physical or human capital (e.g., Lucas (1988), Rebelo (1991) and Romer (1986)). On the other, there are the technology-based models that take the endogenous innovations resulting from advances in technology as the primary source of growth (e.g., Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom, Anant, and Dinopoulos (1990)). Whereas previous research efforts focused exclusively on technological progress as the main engine of growth, recent models show that growth can be self-sustaining without technological progress (Lucas (1988)). In fact, in many of these "*new growth theory*" models, capital accumulation has the ability to increase growth through a number of different channels (Pagano (1993)).

Traditionally, economic growth and business cycle theory have been treated as unrelated[2] areas in macroeconomics (see Lucas (1987), Stiglitz (1993)[3]). Groundbreaking work by Nelson and Plosser (1982), Kydland and Prescot (1982, 1990) and a seminal paper by Ramey and Ramey[4] (1995) have worked to change this perspective with evidence on the linkages between these two fields of economics. Badinger (2009) in a recently published paper argues that while

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[2] In fact, this led to Lucas' (1987) position that the benefits from an understanding of the business cycle we trivial as compared to those from growth.

[3] Stiglitz (1993) argues that a problem with the estimation of the effect of volatility on growth is the causality effect, where causality may also run from growth to volatility

[4] The empirical results of the seminal paper by Ramey and Ramey (1995) suggest a negative net effect between the business cycle and economic growth.

there is still no clear answer on how output volatility affects economic growth the emerging strand of literature is bifurcated into two general areas. The first suggests a positive association such as those in Caballero and Hammour (1991), Hall (1991) and Imbs (2007). Whilst the others advocate a negative relation arising from the presence of irreversibility or diminishing returns to investment or from credit market imperfections that constrain investments during recessions (see for example Aghion and Howitt (2006)).

Recent events in global credit markets may hold further empirical insights on the linkages put forward by Ramey *et al* (1995) and Badinger (2009) and further bolster the viewpoint that households face substantial amounts of business cycle risks in capital accumulation (Borensztein and Lee (2002) and Yellen (2008)). These empirical observations have important economic implications. Building on the work of Imbs, (2007), Aghion and Howitt, (2006), Ramey and Ramey (1995) and Badinger (2009) we argue that business cycle risks faced in the pursuit of capital accumulation may generate precautionary savings if households are prudent, but if households are risk averse this may affect their incentives to invest in human and physical capital thereby negatively impacting economic growth. This is consistent with Federal Reserve data on the 2007-2008 financial crises that shows investors shifting from riskier investments such as equity to safer products such as certificates of deposit and U.S. Treasury backed money market mutual funds[5]. This shift in investment behavior was confirmed in a concurrent survey by the American Association of Individual Investors (AAII) at the end of March 2009 which suggested that individual investors have shifted away from stocks, with just 54% of their money in stocks compared to the long-term average of 60%[6].

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[5] Baba *et al* (2009) provided empirical evidence which showed between the periods late 2007 to April 2008 investors strongly favored government funds over prime risky MMK funds (see figures 2 and 3 in Baba *et al*).
[6] Borensztein and Lee (2002) also found similar evidence in data on Korea's 1998 financial crisis and credit crunch.

Although the effects of capital accumulation and technological change on economic growth have been analyzed extensively, the corresponding effects of business cycle risk on capital accumulation and economic growth has received little attention[7]. As such this paper develops a tractable dynamic optimization model of economic growth in which households are allowed to invest in both risk-free assets and risky capital assets. By combining these two strands of literature, this paper examines the interplay between business cycle risk, capital accumulation and economic output. Moreover, given the emerging empirical evidence (Ramey *et al* (1995 and Badinger (2009)) on the implications of business cycle volatility on output growth this paper considers the capital accumulation process to be defined by the risk-return process as defined in modern portfolio theory. We introduce separate volatility measures of business cycle risk (credit and market risk) so as to capture the dynamics of both forces in financial markets. Both processes are assumed independent. While investors are capable of hedging away the systematic market risk component, non-systematic credit events are much more difficult to hedge (Shultz (2008)). Moreover, with both components in the proposed dynamic growth model we can more easily simulate the effects of a change in one relative to the other on output growth.

Definition 1.0: (a) *Market risk is the systematic change in household's capital stock portfolio (wealth). These changes are as a result of volatilities in market activity and can be hedged away by diversifying the investment portfolio.*

(b) *Credit risk is influenced by both business cycles and firm-specific events. Credit risk typically declines (rises) during economic expansions (contractions) because strong (deteriorating) earnings keep overall default rates low (high). Credit risk is measured by changes in the credit default swap (CDS) on corporate investments. Increases in the CDS spreads also acts as a proxy of rising investor aversion to potential default risk within their portfolios.*

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[7] From their empirical work on the links between business cycle volatility and economic growth, Ramey and Ramey (1995) found compelling evidence of a link between both variables and argued that it would be inaccurate to assume otherwise. They suggested that an omission of these links could lead to questionable conclusions such as those which led Lucas (1987) to underestimate the potential benefits of the linkages between business cycle and output.

As we have pointed out earlier, since the 1980's there has been a growth in real business cycle research examining the cyclical behavior of macroeconomic quantities. Ramey *et al* (1995) extended the earlier work of Nelson *et al* (1982) and Kydland *et al* (1982, 1990) with the publication of an empirical study illustrating the link between business cycle volatilities and growth. Prior to this, a paper by King *et al* (1988) incorporating endogenous growth in a real business cycle model illustrated that temporary disturbances to the various channels of growth can have a permanent effect on output. In fact, earlier work by Friedman and Schwartz (1963) suggest that the negative correlation between economic growth and credit risk is critical to understanding the severity of any financial crisis. Later papers including those of Acharya, Bharath and Sirivaesan (2007) and Duffie, Saita and Wang (2007) show that credit risk as evidenced by default probabilities vary across business cycles. Like the data depicted in figure 1, Tang and Yan (2006) have provided empirical evidence that shows that credit risk is countercyclical with economic growth; widening during recessions and narrowing during an economic expansion.

Rebelo (2005) raised the difficult question of what types of shocks are responsible for business cycle fluctuations. The literature suggests monetary, price, technology, fiscal and oil shocks to list a few[8]. From figure 1 we see that credit risk which is investment in nature and market risk which originates from monetary policy appear to negatively impact output growth. An observation which appears consistent with the strand of literature that argues that business cycle risk is countercyclical to output growth. Motivated by the evidence of an inverse correlation between economic growth and business cycle volatilities (Ramey *et al* (1995 and Badinger (2009), our aim is to develop an understanding of the risk-return profile of the capital

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[8] For added insights the reader may see Real Business Cycle (RBC) research by Rebelo (2005) who provides an historical review of the literature, Barsky and Killian (2004), Christiano and Eichenbaum (2004), Bernanke, Gertler, and Gilchrist (1999).

accumulation process of economic growth. Recent empirical evidence on the United States economy presented in figure 1 suggests that unsystematic shocks that occur because of idiosyncratic credit events in financial markets may cause volatile business cycles. In particular, when unsystematic shocks and aggregate economic output shift and becomes highly correlated, then the economy may be at risk of experiencing undesirable economic fluctuations[9].

Figure 1: For the priod Sept-05 to Dec-08, the graph illustrates the co-movements of credit risk, market risk, networth and economic growth using high frequency daily data.
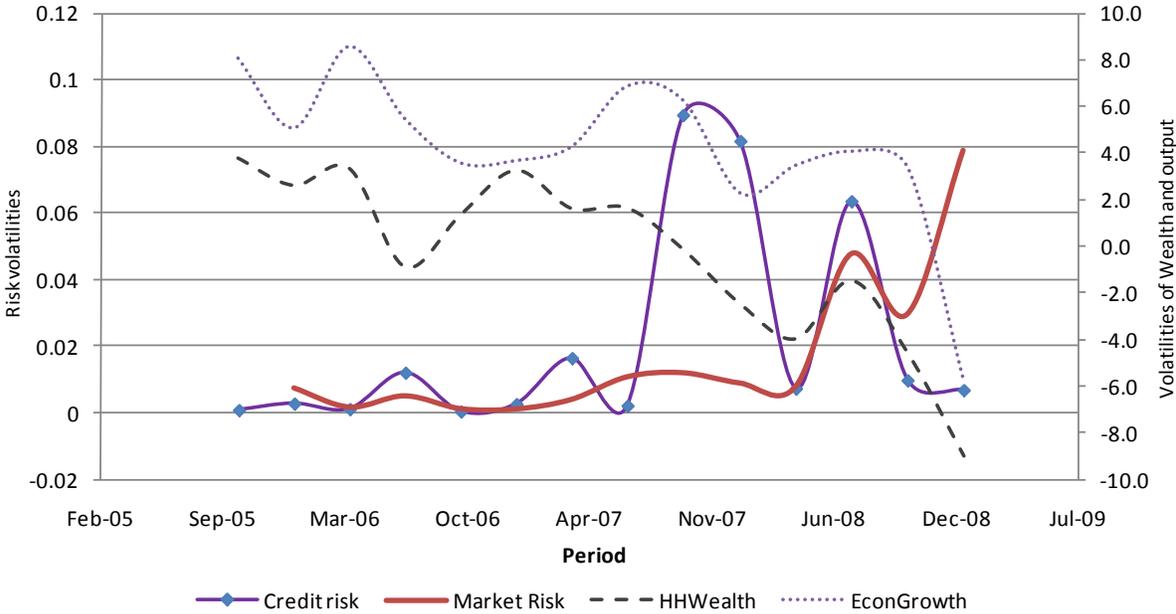
Note: Between the period May-07 and May-08 market risk alone did not clearly indicate the magnitude of business cycle risk faced by households. During this period several companies including Lehman Brothers and Bear Sterns filed for bankruptcy and there were several major debt write-downs by a number of firms, see Crouhy *et al* (2008).
Source: Feds Quarterly Flow of Funds, Federal Reserve Bank of St Louis and Bloomberg.

The recent experiences and consequences of credit market failure [10] have clearly illustrated the negative effects of stochastic shocks on the capital accumulation process. This is

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[9] See Friedman and Schwartz (1963, p.312). The negative correlations between growth and the yield on credit risky bonds are crucial to understanding the severity of a financial crisis.

[10] An underperforming financial system leads to an inadequate channeling of funds from savers to borrowers, where the latter have productive uses at hand but lack the necessary funds. This leads to low rates of accumulation of physical and human capital.

because economic growth in a modern economy hinges on an efficient and stable credit market that works to accumulate domestic savings and attract foreign capital for the productive sector[11]. In fact, Yellen (2008) suggested that,

> *"if anyone ever needed a demonstration on the strength of the links between the functioning of the financial system and the functioning of the economy, then this is it....where a genuine crisis in financial markets, has generated a severe credit crunch. The credit crunch in turn has left households and firms with fewer resources to finance spending, and as a result, output growth weakened"*.

During the credit market crisis it was observed that aggregate household wealth fell by a record $11.2 trillion[12] (-9%) as home values and stock prices deflated. This decline in household wealth slowed capital accumulation and subsequently economic growth (-6.4%). The situation was further compounded by the attempts of banks to preserve their regulatory capital ratios, which saw them cutting off the flow of credit, resulting in a decline in lending to companies and consumers, further acerbating the downturn in economic output. This subsequently led a number of economists to suggest that tighter credit conditions would knock 1.25 percentage points from U.S. first-quarter 2008 growth and 2.5 points from second-quarter 2008 growth (see Crouhy *et al* (2008)).

The model differs from previous theoretical models of economic growth by focusing on the risk-return relationship of capital accumulation. The main goal of the present paper is to endogenously model and illustrate the impacts of business cycle risk on capital accumulation which should then addresses the following questions for a representative open economy in a dynamic model; (i) What is the effect of a stochastic volatility shock on the components in total capital stock and growth, (ii) How does households risk-aversion choices affect productive capital stock creation, precautionary savings, capital accumulation and economic output, (iii)

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[11] See Bernanke *et al* (1999) for a discussion on the financial accelerator.
[12] The Fed's Quarterly Flow of Funds Report 4Q08

Does the Solow (1956, 1957) growth model's assumption of a constant capital stock hold true for the new growth model, where we introduce risk and household risk aversion choices. By addressing these questions we are able to provide an explanation that would help to explain variations in economic growth across economies during the 2007-2008 credit market crises. In other words, we are able to account for the fact that economies with different levels of credit market volatilities will react differently in accumulating capital.

The remainder of the paper is organized into five sections. In section 2 we formulate the capital accumulation optimization problem and lay out the basic setup of the model investigated in this paper. This section introduces the stochastic optimal control growth framework. Section 3 develops a stochastic optimal growth problem inclusive of business cycle risk and derives the optimal share of productive capital and economic growth given output volatilities. Section 4 presents comparative dynamics involving some representative calculations and the main results of the paper. We compare the solutions to that of real world observation and prior research findings. Section 5 summarizes the study's finding and proposes areas of future research.

2.0   The Model Framework: Capital Accumulation under Stochastic Financial Risk

The general model framework described in this section makes use of two generally accepted analytical procedures to derive various aspects of the model. The variance sum law is first used to derive a weighted measure of financial (market and credit) risk in the economy and stochastic optimal control is later used to derive the optimal share of productive capital in the home country's total capital stock and mean level of economic growth. The paper follows the traditional stochastic dynamic programming technique (Merton (1969, 1971), Fleming and Stein

9

(2004) and Gong and Zou (2003)) leading to the Hamilton-Jacobi-Bellman (HJB) equation (Øksendal, 2000; and Björk, 1998 offer a complete derivation of the HJB equation).

Production Function

The proposed stochastic economic growth model developed hereafter describing capital accumulation and economic output is an extension of the classic Solow (1956) model. We begin with the supposition that some final consumption good Y is produced by combining capital and labor, which from the conventional production function,

$$Y = A(\lambda_Y K)^\alpha (\mu_Y L)^{1-\alpha} \tag{1}$$

Where we follow the common practice of letting A denote the exogenous level of technology, K denotes the stock of productive capital, $\mu_Y \in [0,1]$ is the fraction of the labor force employed in the production of final goods, L is the aggregate stock of labor that grows at some rate n, and $\alpha \in (0,1)$ is capital's share of aggregate output. The model assumes fixed coefficient of technology (A) with embodies technical progress (An abundance of research literature suggests that technology change has remained stagnant since the 1970s).

Our capital accumulation equation resembles that of the Solow model,

$$K = I - \delta K \tag{2}$$

Where I denotes total available capital for investment, and δ is the rate at which capital depreciates.

However, whereas the Solow (1956) model assumes that investment always equals savings, we argue that this is not always the case. At the height of the recent credit crisis in early 2008, the U.S. treasury and the central bank increased credit availability to banks to the tune of approximately $450 Billion which was also accompanied by an increase in the U.S. savings

rate[13], a move from less than 1% to more than 4%. These increases in the capital stock did not translate into an equally concurrent increase in lending to the productive sector. To the very extreme banks were accused of hoarding cash and not lending to investors as banks shore-up their regulatory capital ratios and liquidity reserves against perceived write-down's. Hence we modify equation 2 of this study to show that the available capital stock from savings is made up as follows

$$k_p + k_n = I \tag{3}$$

where the productive investment component of capital stock is represented as k_p, and where k_n represents the non productive component of the nation's capital stock that is invested in risk-free government debt because of rising investor aversion to corporate risk. Such a split is not entirely novel, as researchers such as Most and Van den Berg (1996) and Roy and Van den Berg (2006) have used this approach to investigate whether the channels of capital accumulation determines the level of economic growth.

To formulate the stochastic control problem associated with the model, we must specify the state and control variables, the constraints, the dynamics of the process and the criterion to be optimized. Following Fleming and Stein (2004) and Gong and Zou (2003) we develop the optimization problem over an infinite horizon of the expectation of the discounted value of the utility of consumption. The inclusion of stochastic market and credit risk elements in the model is a natural outgrowth of recent quantitative advances in modeling output growth in stochastic environments in Eaton (1981), Gertler and Grinols (1982), Pindyck and Solimano (1993), Turnovsky (1993, 2000), Ramey and Ramey (1995) and Obstfeld (1994).

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[13] Household saving is the primary domestic source of funds to finance capital investment, which is a major stimulus for long-term economic growth.

Business Cycle Volatilities

As in Fleming and Stein (2004) we assume a small open economy and following Eaton (1981) and Turnovsky (2000), the change in output is produced by a stochastic technology

$$\partial Y = F(k_p)\partial t + H(k_p)\partial W \qquad F'(k_p) > 0, \ F''(k_p) < 0 \qquad (4)$$

Where equation 4 asserts that the flow of output over some period $(t, t+\partial t)$ consists of two components, a deterministic component $F(k_p)\partial t$ which represents the mean rate of output production per unit of time and $H(k_p)\partial W$ that represents a generalized stochastic component or independent set of Wiener processes of the various random elements affecting output production. It is assumed that credit and market risk comprise this stochastic space. The independent stochastic processes of market $(\mathcal{Y}_r)$ risk may be represented as

$$\partial \mathcal{Y}_r = \varpi \mathcal{Y}_r \partial t + \sigma_{\mathcal{Y}_r}^2 \partial y \qquad (5)$$

Where the deterministic component of market risk is $\varpi \mathcal{Y}_r \partial t$, and the stochastic component is represented as $\sigma_{\mathcal{Y}_r}^2 \partial y$.

We assume that the credit risk $(\mathbb{C}_r)$ process can be described by a stochastic differential equation with (possibly asymmetric) random jumps[14] such as Das (1999):

$$\partial \mathbb{C}_r = \kappa (\bar{\mathbb{C}}_r - \mathbb{C}_r)\partial t + \sigma_{\mathbb{C}_r}^2 \partial z + \varsigma(t)\partial J \lambda \qquad (6)$$

Where ς is a random jump whose size has a lognormal distribution with constant mean and volatility, and the arrival of jumps is governed by a Poisson process with arrival frequency parameter λ. This parameter indicates the number of jumps per year. The diffusion and Poisson process are independent of each other and independent of ς as well. The return evolves with a

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[14] We use a jump diffusion model for credit risk because jumps in the risk of default which are brought on by deteriorating macro economic conditions can lead to the sudden collapse (default) of the firm.

mean-reverting drift and two random terms, one is Brownian diffusion and the other a Poisson process with random jumps ς. The stochastic terms ∂y and ∂z in equations 5 and 6 are assumed to be temporarily independent, normally distributed with mean zero and variances $\sigma_y^2 \partial t$ and $\sigma_{\mathbb{C}_r}^2 \partial t$, respectively (Turnovsky (2000) provides further insights and justification). The covariance of the stochastic terms is represented as $Cov(\partial y, \partial z) = \sigma_{yz} \partial t$.

2.1   Business Cycle Volatilities: Combining market and Credit Risk

The study draws upon the "variance sum law" to develop a proxy risk measure of *total business cycle risk* $\left(\sigma_B^2 = \left(\sigma_y^2 + \sigma_{\mathbb{C}_r}^2\right)\right)$ that is used to determine the effect of risk on economic growth. From the "variance sum law" we add the variance of systematic market risk [15] $\left(\sigma_y^2\right)$ which is conditioned on changes in household wealth and credit risk $\left(\sigma_{\mathbb{C}_r}^2\right)$ derived from investors' aversion to changes in the credit riskiness[16] of firms and sovereign investments. The approach allows us to add a greater level dynamism to the model whereby changes can be made in one variable relative to the other. The "variance sum law[17]" indicates that for any two random independent samples, the variance of their sum can be found by adding their respective variances which yields;

$$\sigma_B^2 = \sigma_y^2 + \sigma_{\mathbb{C}_r}^2 + 2\rho\sigma_y\sigma_{\mathbb{C}_r} \tag{7}$$

If the two samples are independent their correlation $\rho = 0$, so the last term drops out leading to expression 8

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[15] Derived from volatility of asset prices or could be proxied by the Chicago Board of Trades VIXX index.
[16] Derived from U.S. Sovereign Credit Default Swaps

[17] In fact, the Variance Sum Law states that: The variance of a sum or difference of two independent shocks $\left(\sigma_{\mathbb{C}_r}^2\right)$ and $\left(\sigma_y^2\right)$ is equal to the sum of their variances $\sigma_B^2 = \left(\sigma_y^2 + \sigma_{\mathbb{C}_r}^2\right)$.

$$\sigma_B^2 = \left(\sigma_y^2 + \sigma_{\mathbb{C}_r}^2 \right) \tag{8}$$

Where $\rho = \dfrac{\mathrm{cov}(\sigma_y^2, \sigma_{\mathbb{C}_r}^2)}{\sigma_y \sigma_{\mathbb{C}_r}} =$ the correlation coefficient (between 1 and -1) describes how a set of

data move together. The symbols $\left(\sigma_y \right)$ and $\left(\sigma_{\mathbb{C}_r} \right)$ are the respective standard deviations of

market and credit risk, and $\rho \sigma_y \sigma_{\mathbb{C}_r}$ in equation 7 above is the covariance of market $\left(\sigma_y^2 \right)$ and

credit risk $\left(\sigma_{\mathbb{C}_r}^2 \right)$.

From the central limit theorem we know that the variance of a sampling distribution

is σ^2 / n, combining with the variance sum law and making an additional assumption called the

"homogeneity of variance", the assumption that the variances in the two respective samples are

equal (with unequal n), then

$$\sigma_B^2 = \frac{(n-1)\sigma_y^2 + (l-1)\sigma_{\mathbb{C}_r}^2}{n+l-2} \tag{9}$$

Where n and l are the number of observations in both sets of risk data and the sample variances

will vary because of sampling variation. The reader may refer to Howell (2008) for further

insights in pooled variances[18].

Table 1 presents statistical estimates of market risk extracted from the S&P-500 using

daily data for the period July 25th 2005 to December 30, 2008. Credit risk estimates for the

corresponding period were estimated from sovereign credit default swap for the United States[19].

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[18] Howell, D. C. (2008), Fundamental statistics for the behavioral sciences (6th Ed.). Belmont, CA: Thomson Wadsworth.

[19] Credit Credit-default swaps (CDS) are derivatives used to hedge against losses or to speculate on the ability of companies to repay their debt. The contracts pay the buyer face value in exchange for the underlying securities if a borrower fails to adhere to its debt agreements, and rise when investor perceptions of credit quality deteriorate. The first credit default swap was introduced in 1995 by JP Morgan, as of 2009, their total value has increased to an estimated $45 trillion to $62 trillion. However, today the total outstanding notional value of credit-default swaps has shrunk by approximately 56 percent to about $27 to $35 trillion. The CDS market has supplanted the bond market as the industry gauge for a borrower's credit quality.

The covariance term in expression 7 was found to be zero so this term fell out leading to expression 8, which is used throughout the rest of the paper to represent households' financial risk on investments.

Table 1: Credit and Market Risk Statistical Estimates

	Stdev	Correlation	Covariance	n
Credit Risk	0.036	0.1	0.000006	832
Market Risk	0.014			

Notes: From equation 7 the last term $(2\rho\sigma_y\sigma_c)$ drops out leading to expression 8. Source of the data is Bloomberg

3.0 Deriving the General Optimality Conditions

As mentioned earlier, the paper follows the traditional stochastic dynamic programming technique (Merton (1969, 1971), Fleming and Stein (2004) and Gong and Zou (2003)) leading to the Hamilton-Jacobi-Bellman (HJB) equation (Øksendal (2000) and Björk (1998) offer a complete derivation of the HJB equation). To solve the dynamic optimization problem, we must introduce a value function, but first, following Gong and Zou (2003) we assume a small open economy where the preferences of the home country is defined on its consumption (c), its productive capital (k_p), and foreign direct investment capital, $k_p^f : u(c, k_p, k_p^f)$. Moreover, suppose the utility function $: u(c, k_p, k_p^f)$ is twice differentiable and concave and suppose the nation chooses its consumption path (c), productive investment capital stock (k_p) and unproductive risk-free capital stock (k_n) to maximize its discounted welfare with a constant discount rate $\rho, (0 < \rho < 1)$ such that,

$$V_x = \max_{\Pi} E_0 \int_0^\infty U(c, k_p) e^{-\rho t} \partial t \tag{10}$$

15

Capital Accumulation

Well subject to the technology in equation 1, the risk components and the initial productive capital stock[20] $k_p(0)$, we then establish a domestic household wealth constraint or the capital accumulation over the period $(t, t+\partial t)$ as,

$$\partial k^d = \partial(Y-c) - k_n \partial t \tag{11}$$

where

$$\partial k^d = (I\partial t - \delta k) - k_n \partial t \tag{12}$$

where the change in domestic capital stock ∂k^d, is the difference between gross investment $I\partial t$, the amount of depreciation, δk and the capital flows out of the productive sector to risk-free investments, $k_n \partial t$. Since households are the owners of the traditional factors of production, land, labor and capital, we assume that household will invest net worth which forms the nation's capital stock $(k_p + k_n)$.

Moreover, given that we have an open economy we assume that economic growth also benefits from the inflow of wealth from foreigners which is represented as

$$\partial k^f = (I\partial t - \delta k) - k_n^f \partial t \tag{13}$$

A slight modification in the flow of foreign wealth k^f gives the following Brownian motion representation

$$\partial k^f = \delta k^f \partial t + \sigma_{k^f} \partial x \tag{14}$$

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[20] Where the initial capital stock is the state variable

Where ∂k^f is the change in foreigner's wealth and where the stochastic term (∂x), like ∂y and ∂z in equations 5 and 6, is assumed to be temporarily independent, normally distributed with mean zero and variance $\sigma_{k^f}^2 \partial t$.

Optimality

Similarly to Gong and Zou (2003) to solve the optimization problem we introduce and define the discounted value function

$$\bar{V}\left(k_p, k_p^f, t\right) \tag{15}$$

where the differential operator is given as

$$L\left(\bar{V}\left(k_p, k_p^f, t\right)\right) = \lim_{dt \to 0} E\left(\frac{\partial v}{\partial t}\right)$$

$$= V_t + V_{k_p}\left(\left(F\left(k_p\right) - c\right) + k_p^f\right) + V_{k_p^f}\theta_{k_p^f}k_p^f + \frac{1}{2}V_{k_p k_p} + \sigma_{yz}H\left(k\right)\sigma_{k_p^f}$$

$$+ \frac{1}{2}V_{k_p k_p}H\left(k_p\right)^2\sigma_y^2 + \frac{1}{2}V_{k_p^f k_p^f} + \sigma_y^2\left(k_p^f\right)^2$$

The first three terms represent the expected current period's return to the home country's capital stock, while the remaining terms represent the opportunity cost of risky capital investments. Now given exponential time discounting the value function can be expressed in the form

$$\bar{V}\left(k_p, k_p^f, t\right) = \bar{R}\left(k_p, k_p^f\right)e^{-\rho t} \tag{16}$$

In addition, we denote the share of non-productive capital in total wealth as

$$q = \frac{\partial k_n}{\partial I} = \frac{\partial k_n}{\partial k_p + \partial k_n} \tag{17}$$

and hypothesize that the home country will select the optimal share of productive capital in its total stock of investable capital $(1-q)$ and the optimal consumption path, $(c(t))$ that maximizes the following expression

$$u\left(c,k_p,k_p^f\right)-\rho\bar{R}+\bar{R}_{k_p}\left(F\left((1-q)k_p\right)-c\right)+\bar{R}_{k_p^f}\theta_{k_p^f}k_p^f$$
$$+\frac{1}{2}\bar{R}_{k_pk_p^f}\sigma_{yz^f}H\left((1-q)k_p\right)k_p^f+\frac{1}{2}\bar{R}_{k_pk_p}H\left((1-q)k_p\right)^2\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)+\frac{1}{2}\bar{R}_{k_p^fk_p^f}\sigma_{k_p^f}^2\left(k_p^f\right)^2 \tag{18}$$

where the home country's marginal conditions (the marginal values of consumption and productive capital at the optimum) for optimization are derived by taking the partial derivatives with respects to c and k and are given as

$$\frac{\partial u\left(c,k_p,k_p^f\right)}{\partial c}=\bar{R}_{k_p}, \tag{19}$$

$$\frac{\partial u\left(c,k_p,k_p^f\right)}{\partial k_p}=-\bar{R}_{k_p}F'\left((1-q)k_p\right)-\frac{1}{2}\bar{R}_{k_pk_p^f}\sigma_{yz^f}H'\left((1-q)k_p\right)k_p^f-\bar{R}_{k_pk_p}H\left((q)k_p\right)H'\left((1-q)k_p\right)\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)=0 \tag{20}$$

From equations 19 and 20, with the substitution of the optimal values for capital's share and consumption path (as the functions of $\bar{R}_{k_p},\bar{R}_{k_pk_p^f},\bar{R}_{k_pk_p}$), the value function must satisfy the following Bellman's equation:

$$u\left(c,k_p,k_p^f\right)-\rho\bar{R}+\bar{R}_{k_p}\left(F\left((1-q)k_p\right)-c\right)+\bar{R}_{k_p^f}\theta_{k_p^f}k_p^f$$
$$+\frac{1}{2}\bar{R}_{k_pk_p^f}\sigma_{yz^f}H\left((1-q)k_p\right)k_p^f+\frac{1}{2}\bar{R}_{k_pk_p}H\left((1-q)k_p\right)^2\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)+\frac{1}{2}\bar{R}_{k_p^fk_p^f}\sigma_{k_p^f}^2\left(k_p^f\right)^2=0 \tag{21}$$

3.1   The Explicit Dynamic Programming Solution to the Optimization Problem

In this section we state the dynamic programming solution, which is derived in section 4.0 and given an economic interpretation in section 5. The state variable k_p is defined in equation

3. It is total capital stock available for investing less capital invested in unproductive risk-free investments. The dynamics of the state variable are expressed in equations (11)-(13).

To derive the explicit solutions for, (i) productive capital's share of total capital stock, (ii) household's optimal consumption path and (iii) the mean level of economic growth we specify a similar utility function to expression 10 and of the form

$$u\left(c,k_p,k_p^f\right)=\frac{c^{1-\xi}}{1-\xi}\left(\frac{k_p}{k_p^f}\right)^{-\eta} \tag{22}$$

where ξ and η satisfies the following conditions: if $0<\xi<1$, then $-1<\eta<0$; if $\xi>1$, then $\eta>0$. These conditions guarantee that the utility function is increasing and concave in the relative wealth ratio of domestic and foreign wealth $\left(k_p\middle/ k_p^f\right)$. Now given the country's production technology and from the utility function in expression 22 we express the value function derived in expression 16 as

$$\bar{R}\left(k_p,k_p^f\right)=\chi k_p^{(1-\xi-\eta)}\left(k_p^f\right)^{\eta} \tag{23}$$

Taking partial derivatives of 23 and substituting in expression 19 and 20 yields

$$\frac{c}{k_p}=\left(\chi(1-\xi-\eta)\right)^{-1/\xi} \tag{24}$$

and,

$$-(1-\xi-\eta)A-\frac{1}{2}(1-\xi-\eta)\eta\sigma_{yz^f}+(1-\xi-\eta)(\xi+\eta)(1-q)\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)=0 \tag{25}$$

Substituting equations 24 and 25 into the Bellman's equation yields

$$\left(\left(\chi(1-\xi-\eta)\right)^{-(1-\xi)/\xi}\right)\frac{k_p^{(1-\xi)}}{1-\xi}\left(\frac{k_p}{k_p^f}\right)^{-\eta}-\rho\chi k_p^{(1-\xi-\eta)}\left(k_p^f\right)^{\eta}+\chi\eta k_p^{(1-\xi-\eta)}\left(k_p^f\right)^{\eta-1}\theta_{k_p^f}k_p^f$$

$$+\chi k_p^{(1-\xi)}\left(\frac{k_p}{k_p^f}\right)^{-\eta}(1-\xi-\eta)\left(A(1-q)-\left(\chi(1-\xi-\eta)\right)^{-\frac{1}{\xi}}\right)+\frac{1}{2}\chi(1-\xi-\eta)\eta(1-q)k_p^{(1-\xi)}\left(\frac{k_p}{k_p^f}\right)^{-\eta}\sigma_{yz^f}$$

$$+\frac{1}{2}(1-\xi-\eta)(\xi+q)\chi(1-q)^2 k_p^{(1-\xi)}\left(\frac{k_p}{k_p^f}\right)^{-\eta}\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)+\frac{1}{2}\eta(\eta+1)\sigma_{w^f}^2\chi k_p^{(1-\xi)}\left(\frac{k_p}{k_p^f}\right)^{-\eta}=0 \quad (26)$$

Simplifying we get

$$\left(\chi(1-\xi-\eta)\right)^{-\frac{1}{\xi}}=\frac{(1-\xi-\eta)\frac{1}{2}\left[(\xi+\eta)(1-q)^2\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)-\eta(1-q)\sigma_{yz^f}-2A(1-q)\right]}{(1-\xi-\eta)\xi\big/(1-\xi)}+\frac{\rho-\eta\theta_{k_p^f}-\frac{1}{2}\eta(\eta+1)\sigma_{k_p^f}^2}{(1-\xi-\eta)\xi\big/(1-\xi)} \quad (27)$$

Substituting into (24), we have the optimal control variable

$$\left(\frac{c}{k_p}\right)^*=\frac{\rho-\eta\theta_{k_p^f}+(1-\xi-\eta)\frac{1}{2}\left[(\xi+\eta)(1-q)^2\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)-\eta(1-q)\sigma_{yz^f}-2A(1-q)\right]-\frac{1}{2}\eta(\eta+1)\sigma_{k_p^f}^2}{(1-\xi-\eta)\xi\big/(1-\xi)} \quad (28)$$

simplifying equation 25 we determine the optimal share of productive capital $\left(k_p^*\right)$ in total capital

stock as

$$1-q=\frac{-A-\frac{1}{2}\eta\sigma_{yz^f}}{(\xi+\eta)\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)} \quad (29)$$

and the mean growth rate of the economy ϕ is denoted

$$\phi=E\left(\frac{dk_p/dt}{k_p}\right)=\left(A(1-q)-\left(\frac{c}{k_p}\right)^*\right) \quad (30)$$

where the transversality condition[21] may be represented as

$$\lim_{t\to\infty}E\left[\kappa k_p^{(1-\xi-\eta)}\left(k_p^f\right)^{-\eta}e^{-\rho t}\right]=0$$

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[21] This is also the positivity of the consumption /wealth ratio

The model shows that when the optimal share of productive capital k_p^* is small, the mean growth

ϕ falls (Note: a small k_p^* result in a large $\left(\dfrac{c}{k_p}\right)^*$ which makes ϕ small or alternatively as

consumption rises (falls) the optimal feedback variable (falls)). Also notice that as credit risk

$\left(\mathbb{C}_r\right)$ rises as we see in figure 1, the share of k_p^* falls while the share of k_n^* increases which

indicates that banks may be hoarding cash or households are reluctant to invest (rising risk

aversion) as needed for capital accumulation and subsequently economic growth (see figures 2

and 3 in Baba *et al*) .

4.0   Comparative Dynamics

In this section, we now examine how the effect of a stochastic shock in financial markets

affects economic growth;

Proposition 1. *A stochastic business cycle shock (credit and market risk) lowers the proportion of productive capital in total capital stock.*

Proof. Using implicit differentiation on equation 29, the effects of a stochastic credit shock to the proportion of total wealth that is allocated to risk free assets is given by,

$$\frac{\partial q}{\partial \sigma_{\mathbb{C}_r}^2} = \frac{A + \frac{1}{2}\eta\sigma_{yz}}{\left(\xi+\eta\right)\left(\sigma_y^2+\sigma_{\mathbb{C}_r}^2\right)^2}$$

Where

$$\frac{\partial q}{\partial \sigma_{\mathbb{C}_r}^2} > 0$$

when $\xi > 1$ and $0 < \eta < 1$ □

The analysis indicate that in an environment denoted by rising market and credit risk

volatilities, the country will allocate its capital in such a way that more of its funds will be in

risk-free k_n^* investments and less to the more risky investments k_p^* in firms[22]. This increase in k_n^* relative to k_p^* indicates the growing risk-averseness of investors. During the recent credit market crisis of 2007/2008, investors shocked by the near collapse of the financial system were less willing to risk money in stocks, preferring safer investments such as certificates of deposit and money market mutual funds (see Baba *et al* (2009)). In fact, data from the Federal Reserve during this period showed significant increases in CDs and U.S. Treasury backed Money Market balances. Moreover, a concurrent survey by the American Association of Individual Investors (AAII) also showed that during this period households were underinvested in stocks, with just 54% of their money in stocks compared to the long-term average of 60%.

Proposition 2. *As the optimal share of productive capital declines, the mean growth rate of the economy also falls.*

Proof. Using implicit differentiation on equation 30,

$$\frac{\partial \phi}{\partial q} = -A - \frac{\frac{1}{2}(1-\xi)(2A+\eta)\sigma_{yz}^2 - 2(\xi+1)(\sigma_y^2 + \sigma_{cr}^2)}{(\xi)}$$

Where

$$\frac{\partial \phi}{\partial q} < 0$$

when $\xi > 1$ and $0 < \eta < 1$ $\square$

As the economy gets less stable as denoted by the relative proportion of capital allocated to risk-free capital k_n^*, the mean growth rate of the economy suffers. The predictions of the model are broadly consistent with the observed investment decisions and economic results seen during the recent global credit crisis and in prior work by Borensztein and Lee (2002), Mishkin (1999) and Baba *et al* (2009).

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[22] Borensztein and Lee (2002) also reported similar empirical results on the 1998 financial crisis and credit crunch in Korea.

Proposition 3. *Stochastic business cycle shocks to output growth increases precautionary savings (capital accumulation), but the home country may allocate more of these resources to its non-productive capital stock.*

Proof. Using implicit differentiation on equation 30 and from propositions 1 and 2, these effects are given by,

$$\frac{\partial k_p / \partial \sigma_{cr}^2}{k_p} = \frac{\dfrac{I\left(A - \dfrac{1}{2}\eta\sigma_{yz}^2\right)}{(\eta+\xi)\left(\sigma_y^2 + \sigma_{cr}^2\right)^2}}{k_p}$$

Since $\eta > 0$ if $\xi > 1$ we obtain

$$\frac{\partial w^d / \partial \sigma_{cr}^2}{w^d} > 0 \quad \square$$

From the analysis, a stochastic business cycle shock to output growth may raise precautionary savings. However, unlike the traditional approaches to economic growth propositions 1 and 2 illustrates that this rise in savings will not translate into automatic investment as in the Solow model but will increase the stock of risk-free government investments and foreign assets (see Baba *et al* (2009)). During this period the economy tends to become more conservative when there is more to lose[23] (see Borensztein (2002) and Yilmaz (2008)).   Higher business cycle volatility (credit and market risk) lowers economic growth through its effect on total capital stock formation.

Proposition 4. *As the mean growth in foreign capital stock gets higher the home country's elasticity of inter-temporal substitution also rises. Similarly, rising volatility in the foreign capital stock raises the home country's inter-temporal substitution even higher.*

Proof. Using implicit differentiation on equations 30 inclusive of 28 and 29, the effects of growth in foreign capital inflows is given by,

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[23] This condition is generally referred to as a "*flight to quality*" in the investment literature and modern portfolio theory. During the 2007/2008 credit market crisis the U.S. savings rate increased for less than 1% to greater than 4%. This however did not translate in higher lending to the productive sector.

$$\frac{\partial \phi}{\partial \theta_{w^f}} = \frac{(1-\xi)\eta}{\xi(1-\xi-\eta)}$$

Since $\eta > 0$ if $\xi > 1$ we obtain

$$\frac{\partial \phi}{\partial \theta_{w^f}} > 0 \qquad \square$$

and in terms of rising volatility in foreign capital stocks,

$$\frac{\partial \phi}{\partial \sigma_{w^f}^2} = \frac{\frac{1}{2}(1-\xi)\eta(\eta+1)}{\xi(1-\xi-\eta)}$$

Since $\eta > 0$ if $\xi > 1$ we obtain

$$\frac{\partial \phi}{\partial \sigma_{w^f}^2} > 0 \qquad \square$$

The term $1/\xi$ measures the home country's elasticity of inter-temporal substitution. From the analysis we see that a rise (fall) in the growth of foreign capital stocks leads to a higher (lower) level of economic growth if the home country's elasticity of inter-temporal substitution in consumption is smaller (larger). Secondly we also deduce from the analysis that if the home country has a higher (lower) elasticity of inter-temporal substitution in consumption then it will react to rising (falling) volatilities in foreign wealth stocks by cutting consumption and investing more of precautionary savings which will ultimately raise economic output. Empirical work by Borensztein and Lee (2002) provided evidence that higher volatilities in foreign capital markets will force households to shun the corporate sector in general for government securities or foreign

assets[24]. In fact empirical work by Badinger (2009) suggested that spillover volatilities are a relevant determinant of output growth and earlier work by Wacziarg (2001) found that a 10% increase in foreign capital inflows results in a 3.2% rise in growth.

5.0   Conclusions

Favorable Macroeconomic conditions will create conditions that lead to a rise in the return on capital which stimulates economic expansion (Fleming and Stein (2004)). However we argue that a rise in business cycle risk inhibits economic growth by restricting the creation of productive capital accumulation. As business cycle risk rises, risk-averse households invest less in the productive sector, choosing the safety of risk-free government bonds. In figure 1, the period April 2007 through May 2008 experienced a significant rise in credit risk; characterized by a large number of business failures and asset write-downs. Data from the U.S. Federal Reserve and other third party providers at this time showed an increase in the demand for CD's and money market instruments relative to equity products, a phenomenon also observed in data on the Korean financial crisis and credit crunch of 1998.

In this paper we modeled the dynamics between business cycle risk, capital accumulation and economic growth, in order to highlight the implication of business cycle risk on economic growth. Propositions 1-4 summarizing our contribution to the literature, illustrate that rising riskiness in financial markets will result in disproportionately more capital flowing to non-productive capital stock, in which case economic growth suffers. Rising business cycle volatilities will increase discretionary savings, however unlike the Solow growth model that

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[24] Baba *et al* (2009) concluded from work examining the flow of funds from US corporate backed money market products (MMK) during the period August 2007 to August 2008 that because of the level of volatility in U.S. financial markets a significant amount of funds flowed from the U.S. to the safety of offshore non-U.S. banks. They also suggested that from late 2007 to April 2008 investors strongly favored government funds over prime risky MMK funds.

suggests that accumulated capital stock is equal to investment our model demonstrates that higher discretionary savings during periods of rising risk-aversion does not translate into more capital for the productive sector but more inflows to the safety of government bonds; thereby hurting economic growth.

Given the model's implied relationships between risk, capital and economic growth it is not strange to see both the negative and positive associations between aggregate out and capital inflows in many cross country time series studies. To single out a few cases, a positive association between foreign capital flows and output growth is found in Borensztein (2002), Testas (2004), Roy et al (2006), Zeng (1997) and Wacziarg (2001) whose empirical work suggests that a 10% rise in foreign capital inflows will result in a 3.2% increase in growth. From proposition 4, we see that a rise in the growth of foreign capital stock will lead to higher economic growth if the home country's elasticity of inter-temporal substitution in consumption is smaller and if the home country has a lower level of business cycle risk relative to foreigners. Moreover, we conclude from the analysis that if the home country has a higher elasticity of inter-temporal substitution in consumption then it will react to rising volatilities in foreign wealth stocks by cutting consumption and investing more of its precautionary savings which will ultimately raise economic output. The significance of this latter result of the model is that it is consistent with the findings of Easterly *et al* (2002) and Ramey *et al* (1995) showing that small state economies which tend to have higher output volatilities generally attract lower levels of FDI and grows at a slower rate relative to the more developed economies with less output volatilities.

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